Applied Combinatorics

by Fred S. Roberts and Barry Tesman

Answers to Selected Exercises[1]

Chapter 10

Section 10.1. No exercises.

Section 10.2.

1(a). (49, 96);

1(b). (65, 88);

1(c). 44, 84, 42, 56, 42, 32, 33, 48;

1(d). 29, 20, 36, 48, 25, 48, 50, 76, 10, 4, 28, 20, 49, 60, 24, 36, 28, 52;

1(e). 37, 56, 32, 20, 59, 80, 66, 92, 41, 52, 33, 48, 30, 36, 43, 56;

1(f). 5, 8, 51, 72, 60, 80, 18, 20, 31, 36, 57, 76, 39, 60, 66, 104;

2(a). 1, 1, 2, 1;

2(b). 1, 0, 1, 1;

2(c). 0, 1, 1, 0;

2(d). 0, 0, 0, 0;

3(a). 1, 1, 1, 1, 1, 1;

3(b). 1, 0, 1, 1, 0, 1;

3(c). 0, 0, 0, 0, 0, 0;

4(a). 1, 1, 1, 1, 3, 3, 3;

4(b). 1, 0, 0, 0, 0, 1, 1;

4(c). 0, 0, 0, 1, 1, 1, 1;

5(a). (8, 14, 14);

5(b). (46, 91, 96);

[1]More solutions to come. Comments/Corrections would be appreciated and should be sent to: Barry Tesman (tesman@dickinson.edu) or Fred Roberts (froberts@dimacs.rutgers.edu).

5(c). (81, 142, 161);

5(d). (69, 117, 115, 71, 119, 157, 79, 138, 147, 36, 64, 90, 47, 89, 93);

5(e). (43, 81, 91, 52, 84, 100, 36, 59, 72, 58, 97, 134, 18, 31, 42, 39, 73, 89, 43, 78, 73);

5(f). (38, 74, 72, 63, 108, 109, 53, 86, 103, 63, 106, 113, 48, 82, 78, 34, 63, 56, 97, 168, 187);

6(a). (28, 54);

6(b). (106, 170);

6(c). 29, 54, 70, 112, 112, 172, 48, 78;

6(d). 81, 124, 60, 96, 16, 30, 67, 110, 33, 50, 77, 118, 91, 144, 33, 54, 21, 38;

6(e). 50, 82, 93, 142, 96, 154, 103, 166, 73, 116, 48, 78, 57, 90, 74, 118;

6(f). 6, 10, 78, 126, 100, 160, 37, 58, 61, 96, 95, 152, 51, 84, 82, 136;

7(a). (9, 8, 8);

7(b). (65, 46, 4);

7(c). (82, 81, 80);

7(d). (73, 69, 84, 53, 71, 92, 84, 79, 80, 29, 36, 32, 61, 47, 20);

7(e). (52, 43, 20, 40, 52, 80, 28, 36, 52, 40, 58, 76, 14, 18, 20, 43, 39, 20, 55, 43, 32);

7(f). (55, 38, 8, 67, 63, 72, 41, 53, 80, 61, 63, 80, 53, 48, 56, 47, 34, 20, 97, 97, 104);

8(a). AB;

8(b). BC;

8(c). error;

8(d). AC;

8(e). error;

8(f). error;

8(g). error;

9(a). ABC;

9(b). error;

9(c). BAT;

9(d). PIRATE;

9(e). error;

10(a). AA;

10(b). error;

10(c). error;

10(d). AC;

10(e). error;

10(f). error;

10(g). error;

11(a). error;

11(b). error;

11(c). error;

11(d). error;

11(e). error;

12(a). **(i)** 011;

12(a). **(ii)** 010;

12(a). **(iIi)** 110;

12(b). **(i)** 1001;

12(b). **(ii)** 0011;

12(b). **(iii)** 1101;

12(c). **(i)** 10000;

12(c). **(ii)** 11001;

12(c). **(iii)** 01110;

13. 101, 000, 011, 011;

14. a double repetition code;

15. using notation from Example 10.3, let **M**$= (I_k \quad I_k \quad \cdots \quad I_k)$ where there are p copies of I_k which is the $k \times k$ identity matrix;

16. using notation from Example 10.4, let

$$\mathbf{M}_{i,j} = \begin{cases} 1 & \text{if } \pi(i) = j \\ 0 & \text{else} \end{cases} .$$

***Section 10.3*.**

1(a). 6;

1(b). 5;

1(c). 2;

1(d). 3;

2(a). $111111\underline{0}$;

2(b). $1001011\underline{0}$;

2(c). $001001001\underline{1}$;

2(d). $01010110111\underline{1}$;

3(a). detect 2, correct 1;

3(b). detect 3, correct 1;

3(c). detect 3, correct 1;

4(a). median: 00000000, mean: 00000000;

4(c). median: 000000000, mean: 000000000;

5(a). 0;

5(b). 1;

5(c). 1;

5(d). 1;

6. For example, let $C = \{001111, 110110\}$ and x_i's: $001000, 110110$;

7(a). (i) 4, **(ii)** 4, **(iii)** 4;

7(b). $d - 1$;

7(c). $\lceil (d/2) - 1 \rceil$;

8. $\frac{2^{10}}{\binom{10}{0}+\binom{10}{1}+\binom{10}{2}+\binom{10}{3}} = 2^{10}/176$;

9. $\frac{2^{11}}{\binom{11}{0}+\binom{11}{1}+\binom{11}{2}} = 2^{11}/67$;

10. $\frac{2^{7}}{\binom{7}{0}+\binom{7}{1}+\binom{7}{2}} = 2^{7}/29$;

11(a). $\binom{10}{0}(.1)^0(.9)^{10}$;

11(b). $\binom{10}{1}(.1)^1(.9)^{9}$;

11(c). $\binom{10}{2}(.1)^2(.9)^8$;

11(d). $1 - \left(\binom{10}{0}(.1)^0(.9)^{10}\right) - \left(\binom{10}{1}(.1)^1(.9)^9\right) - \left(\binom{10}{2}(.1)^2(.9)^8\right)$;

12(a). $\binom{6}{0}(.001)^0(.999)^6$;

12(b). $\binom{6}{1}(.001)^1(.999)^5$;

12(c). $\binom{6}{2}(.001)^2(.999)^4$;

12(d). $1 - \left(\binom{6}{0}(.001)^0(.999)^6\right) - \left(\binom{6}{1}(.001)^1(.999)^5\right) - \left(\binom{6}{2}(.001)^2(.999)^4\right)$;

13(a). $\binom{n}{t}(p)^t(1-p)^{n-t}$;

13(b). $\left[\binom{5}{5}(p)^5(1-p)^0\right] \cdot \left[\frac{1}{2^5}\right]$;

13(c). $\left[\binom{4}{3}(p)^3(1-p)^1\right] \cdot \left[\frac{1}{\binom{4}{3}3^3}\right]$;

14. probability of 0 errors is .729, of 0 or 1 errors is .972, so $d = 3$;

19(b). 1 error;

22(b). 2 errors (use horizontal completeness).

Section 10.4.

1(a). 1101;

1(b). 1011;

1(c). 0110;

1(d). 0000;

2(a). 111111;

2(b). 101101;

2(c). 000000;

3(a). 1111111;

3(b). 1000011;

3(c). 0001111;

4(b). the $3 \to 6$ double repetition code;

5. $\mathrm{M} = \left[\begin{array}{cccccc} 1 & 1 & 1 & 1 & 1 & 1 \end{array}\right]$;

6(a). 2;

7(a). $\left(\begin{array}{cccc} 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{array}\right)$;

8(a). $\begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$;

9(a). $x_3 = x_1 + x_2, x_4 = x_1$;

11(a). no error;

11(b). yes – an error;

12(c). yes – an error;

14(b). 111000;

15(c). not possible;

17(a). 1001100;

21. 111 if $p = 2$;

27. use $|C| = 2^{2^p-1-p}$, $t = 1$;

28(a). (i) 1401;

28(b). (ii) 102102;

29(b). $d(\mathbf{x}, \mathbf{y}) = wt(\mathbf{x} - \mathbf{y})$.

Section 10.5.

2(e). $2(r - \lambda) - 1$;

4(a). no;

5. yes;

9(b). 7;

16(a). $2m$;

16(b). $4m - 1$ if $i = j, 2m - 1$ otherwise;

22(b). no;

26. $\mathbf{M} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$;

29(a). the distance between codewords of equal weight is even.