Applied Combinatorics

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Answers to Selected Exercises[1]

Chapter 11

***Section 11.1*.**

1(a). One example: assign labels $a = 1, e = 2, j = 3, k = 4, f = 5, g = 6, h = 7, l = 8, m = 9, i = 10, d = 11, c = 12, b = 13$ and mark the edges $\{a, e\}, \{e, j\}, \{j, k\}, \{k, f\}, \{f, g\}, \{g, h\}, \{h, l\}, \{l, m\}, \{m, i\}, \{i, d\}, \{d, c\}, \{c, b\}$;

2. not connected;

3(a). connected;

4(a). one example: use the marked edges in answer to 1(a);

6. it is a spanning forest;

8(a). no;

10. yes.

***Section 11.2*.**

1(a). none;

1(f). $\{c, e\}$;

2(a). orientation based on answer to Exercise 1(a), Section 11.1 orients 1 to 2 to 3 to ... to 13 and all other edges from higher number to lower number;

7. $V = \{x, a, b, c\}, E = \{\{x, a\}, \{x.b\}, \{x, c\}\}$;

8(a). none;

8(f). c, e;

16. for digraph (a) and measure (1): essentially the only orientations are: (i) which uses arcs $(a, b), (b, c), (c, f), (f, e), (e, d), (d, a)$, and (b, e), or (ii) which uses arcs $(a, b), (b, e), (e, d), (d, a), (e, f), (f, c)$, and (c, b); both are equally efficient; for digraph (a) and measure (3): orientation (ii) above is best; for digraph (a) and

[1] More solutions to come. Comments/Corrections would be appreciated and should be sent to: Barry Tesman (tesman@dickinson.edu) or Fred Roberts (froberts@dimacs.rutgers.edu).

measure (4): orientation (ii) above is best; for digraph (a) and measure (6): orientation (ii) above is best;

25. D_1: category 3, D_4: category 2;

26. if $V = \{a, b, c, d\}$ and $A = \{(a, b), (b, c), (c, d), (d, a)\}$, any arc is (3, 2);

27. in previous example, any vertex is (3, 2);

28(a). 1.

***Section 11.3*.**

2. G_1 : none; $G_2 : a, c, e, b, f, e, i, j, k, l, i, k, h, g, f, d, a$; G_3 : none; $G_4 : f, e, c, d, e, b, a, b, a, d, g, f$;

3. G_1;

4. D_1: $a, b, d, c, d, f, e, c, a$; D_2: none; D_3: none; D_4: none; D_5: none;

5. $D_2 : c, a, b, d, f, e, c, d$; $D_5 : b, a, e, f, g, b, c, d, e$;

10(a). yes;

10(b). yes;

10(c). no;

10(d). yes;

12(a). $D_1 : 2$;

12(b). $D_2 : 2$; $D_5 : 2$;

13. 12;

16. no: consider D_2 of Figure 11.25.

***Section 11.4*.**

3(a). add edge $\{c, f\}$;

3(b). add edges $\{e, g\}$ and $\{g, h\}$;

3(c). add edges $\{a, d\}$ and $\{b, c\}$;

3(d). add edges $\{a, d\}$ and $\{b, c\}$;

6(a). CAAGCUGGUC;

9(a). yes: $A_1A_1A_2A_2A_2A_3A_3A_3A_3A_3A_2A_1$;

17(a). say B is an interior extended base of a U, C (G) fragment; then both B and the preceeding extended base end in G (U, C), so B is on the second list;

18. ends in A;

20(b). if there is a second abnormal fragment, it is B alone.

Section 11.5.

1(a). a, b, d, f, e, c, a;

1(b). $i, a, b, c, d, e, f, g, h, i$;

1(c). $a, b, c, d, h, g, f, j, k, l, p, o, n, m, i, e, a$;

2(a). e, c, d, a, b;

2(b). a, b, c, d, f, e;

2(c). $a, b, c, d, e, j, g, i, f, h$;

3(a). a, b, c, d, e, f, a;

3(b). a, b, c, e, d, a;

3(c). c, a, d, b, e, f, c;

4(a). a, d, b, c;

4(b). a, b, d, c, e, f;

4(c). a, c, d, b, e;

6. no;

7(a). Z_4;

7(b). K_4;

7(c). the graph in Figure 11.50;

7(d). $K_{1,4}$;

8(a). $K_{1,3}$;

8(b). K_4;

9(a). yes;

10(a). for (a) of Figure 11.45: complete graph;

10(b). for (a) of Figure 11.45: yes;

11. $K_{1,3}$;

12. Z_5;

13. for (a) of Figure 11.4: no;

14. for (a) of Figure 11.4: no;

15(a). use Theorem 11.8.

Section 11.6.

1(b). yes;

2. for (b) of Exercise 1, Section 4.1: no;

3. for (a), the labeling is a topological order;

8. SF, B, H, LA, NY or SF, B, LA, NY H, or SF, B, NY H, LA;

10. i beats j iff $i < j$;

11(a). $a = 1, b = 2, c = 3, d = 4, e = 5, f = 6$;

12. 3, 1, 2;

15. if C is a, b, f, d, c, e, a, then H has edges $\{b, d\}$ to $\{e, f\}$ and $\{a, c\}$ to $\{e, f\}$ and is 2-colorable;

20(b). (0, 1, 2, 3, 4);

22(a). no;

22(b). no;

23. consider the tournament on $V(D) = \{1, 2, 3, 4\}$ and $A(D) = \{(1, 2), (2, 4), (3, 1), (3, 2), (4, 1), (4, 3)\}$;

28(a). $(0, 1, 2, \ldots, n - 1)$;

31. start with any vertex x and find the longest simple path heading into x; this must start at a vertex with no incoming arcs;

32(a). $\binom{s(u)}{2}$;

32(c). use part (b) and the fact that $s(u) \geq 2$ for some vertex u in every tournament of four or more vertices.