Applied Combinatorics

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Answers to Selected Exercises[1]

Chapter 2

***Section 2.1*.**

1. yes: $26^3 < 20,000$;

2. yes: $26^3 \cdot 10^3 > 5,000$;

3(a). $3^1 + 3^2 + 3^3$;

3(b). $3^1 + 3^2 + 3^3 + 3^4$;

3(c). $2 \cdot 3^3$;

4. $8^3 \times 10^5; 8 \times 2 \times 10 \times 8^3 \times 10^5$;

5. $n = 5$ since $2^1 + 2^2 + 2^3 + 2^4 = 30$ and $2^1 + 2^2 + 2^3 + 2^4 + 2^5 = 62$.

6. 2^{mn};

7. $(2 \cdot 2 \cdot 2 \cdot 2 \cdot 3) - 1$;

8. $10^6 - 9^6$;

9.

Bit string x	$S_1(x)$	$S_2(x)$	$S_3(x)$	$S_4(x)$	$S_5(x)$	$S_6(x)$	$S_7(x)$	$S_8(x)$
00	0	0	0	0	0	0	0	0
10	0	0	0	0	1	1	1	1
10	0	0	1	1	0	0	1	1
11	0	1	0	1	0	1	0	1
Bit string x	$S_9(x)$	$S_{10}(x)$	$S_{11}(x)$	$S_{12}(x)$	$S_{13}(x)$	$S_{14}(x)$	$S_{15}(x)$	$S_{16}(x)$
00	1	1	1	1	1	1	1	1
10	0	0	0	0	1	1	1	1
10	0	0	1	1	0	0	1	1
11	0	1	0	1	0	1	0	1

10. $2^{2^{2^n}}$

[1] More solutions to come. Comments/Corrections would be appreciated and should be sent to: Barry Tesman (tesman@dickinson.edu) or Fred Roberts (froberts@dimacs.rutgers.edu).

11. $2^{2^{n-1}}$;

12. 2^p; $2^p - 1$;

13(a). $2^m 2^{p-m}$;

13(b). $2^{m-1}2^{p-m}$;

13(c). $(2^m - 1) \cdot 2^{p-m}$;

14(a). $3 \cdot 2 \cdot 3$;

14(b). $2^{3 \cdot 2 \cdot 3}$.

Section 2.2.

1. $2^3 + 2^4 + 2^5$;

2. $(8 \cdot 7) + (8 \cdot 12) + (7 \cdot 12)$;

3. $5^5 + 4^5$;

4. Yes: $10^{15} > 10,000,000$; No: $2^{15} < 10,000,000$;

5. $26^4 + 25^5$;

6. $2 + 3$;

7. $(7 \cdot 3)^{30}$;

8. $3^3 + 3 \cdot 4^3$.

Section 2.3.

1(a). 123, 132, 213, 231, 312, 321;

1(b). 1234, 1243, 1324, 1342, 1423, 1432, 2134, 2143, 2314, 2341, 2413, 2431, 3124, 3142, 3214, 3241, 3412, 3421, 4123, 4132, 4213, 4231, 4312, 4321;

2. $1 \cdot 4 \cdot 3 \cdot 2 \cdot 1$;

3. $1 \cdot n - 2 \cdot n - 3 \cdot \cdots \cdot 2 \cdot 1 \cdot 1$;

4(b). $s_6 \approx 710$ vs. $6! = 720$;

5. $2 \cdot 3 \cdot 2 \cdot 1 = 12$;

6(a). $4 \cdot 1 \cdot 3 \cdot 2 \cdot 1$;

6(b). $n - 2 \cdot 1 \cdot 1 \cdot n - 3 \cdot n - 4 \cdot \cdots \cdot 2 \cdot 1$;

7. $1 \cdot 3 \cdot 2 \cdot 1 \cdot 1$;

8(a). $3 \cdot 3 \cdot 2 \cdot 2 \cdot 1 \cdot 1$;

8(a). $n \cdot n \cdot n - 1 \cdot n - 1 \cdot \cdots \cdot 2 \cdot 2 \cdot 1 \cdot 1$;

9. $5! \cdot 5!$ vs. $10!$.

***Section 2.4*.**

1. $25! \cdot \frac{1}{10^9} \cdot \frac{1}{3.15 \times 10^7} \approx 4.9 \times 10^8$ years.

2. $25! \cdot \frac{1}{10^{11}} \cdot \frac{1}{3.15 \times 10^7} \approx 4.9 \times 10^6$ years.

3. There are $n!$ schedules. Each committee in each schedule must be checked to see if it received its first choice - this takes n steps per schedule. The computational complexity is $f(n) = n \cdot n!$;

5. We will start (and end) at 1.

Order	Total cost
1 2 3 4 1	1+ 3+ 11+8 = 23
1 2 4 3 1	1+ 6+ 2+ 4 = 13
1 3 2 4 1	8+ 9+ 6+ 8 = 31
1 3 4 2 1	8+ 11+ 3+ 16= 38
1 4 2 3 1	11+ 3+ 3+ 4 = 21
1 4 3 2 1	11+ 2+ 9+ 16= 38

6. best order is 2, 3, 1;

7(a). $n \times 3 \times 10^{-9}$.

7(b). $\frac{n+1}{2} \times 3 \times 10^{-9}$.

8(a). $n \times 3 \times 10^{-11}$.

8(b). $\frac{n+1}{2} \times 3 \times 10^{-11}$.

***Section 2.5*.**

1(a). $3 \cdot 2$;

1(b). $5 \cdot 4 \cdot 3$;

1(c). $8 \cdot 7 \cdot 6 \cdot 5 \cdot 4$;

1(d). 0;

2(a). 6^3;

2(b). $6 \cdot 5 \cdot 4$;

2(c). $1 \cdot 6 \cdot 6$;

2(d). $1 \cdot 5 \cdot 4$;

3(a). 8^4;

3(b). $8 \cdot 7 \cdot 6 \cdot 5$;

3(c). $1 \cdot 8 \cdot 8 \cdot 8$;

3(d). $1 \cdot 6 \cdot 5 \cdot 1$;

4. $P(20, 5)$;

5(a). $9 \times 9 \times 8 \times 7$;

5(b). 4088. There are $1 \times 9 \times 8 \times 7$ extensions if the first digit is 1 and there are $8 \times 8 \times 8 \times 7$ extensions if the first digit is 2 - 9;

6. $40^3 = 64,000$.

Section 2.6.

1. $\{\}$, $\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$, $\{a,b\}$, $\{a,c\}$, $\{a,d\}$, $\{b,c\}$, $\{b,d\}$, $\{c,d\}$, $\{a,b,c\}$, $\{a,b,d\}$, $\{a,c,d\}$, $\{b,c,d\}$, $\{a,b,c,d\}$;

2. 2^{35};

3. 2^7;

5. $2^{10} - 1$;

6. $2^8 - 9$;

7(a). 2^8. There are $2^3 = 8$ subsets of A and each subset can be assigned a 0 or 1;

7(b). 2^{2^n};

8(a). 2^{2^3};

8(b). 2^{2^n}.

Section 2.7.

1. $C(10, 5)$;

2. $C(50, 7)$;

3(a). $\frac{6!}{3!(6-3)!} = 20$;

3(b). $\frac{7!}{4!3!} = 35$;

3(c). $\frac{5!}{1!4!} = 5$;

3(d). 0;

4. $\frac{n!}{1!(n-1)!} = n$;

5. $\frac{5!}{2!3!} = 10$; $\{a,b\}, \{a,c\}, \{a,d\}, \{a,e\}, \{b,c\}, \{b,d\}, \{b,e\}, \{c,d\}, \{c,e\}, \{d,e\}$;

6. $\frac{6!}{2!4!} = 15$; $\{a,b\}$, $\{a,c\}$, $\{a,d\}$, $\{a,e\}$, $\{a,f\}$, $\{b,c\}$, $\{b,d\}$, $\{b,e\}$, $\{b,f\}$, $\{c,d\}$, $\{c,e\}$, $\{c,f\}$, $\{d,e\}$, $\{d,f\}$, $\{e,f\}$;

7(a). $C(7,2) = \frac{7!}{2!(7-2)!} = \frac{7!}{2!5!}$ and $C(7,5) = \frac{7!}{5!(7-5)!} = \frac{7!}{5!2!}$;

7(b). $C(6,4) = \frac{6!}{4!(6-4)!} = \frac{6!}{4!2!}$ and $C(6,2) = \frac{6!}{2!(6-2)!} = \frac{6!}{2!4!}$;

8. 1 7 21 35 35 21 7 1;

9. $C(5,3) = \frac{5!}{3!2!} = 10$, $C(4,2) = \frac{4!}{2!2!} = 6$, $C(4,3) = \frac{4!}{3!1!} = 4$, and $10 = 6 + 4$;

10. $C(7,5) = \frac{7!}{5!2!} = 21$, $C(6,4) = \frac{6!}{4!2!} = 15$, $C(6,5) = \frac{6!}{5!1!} = 6$, and $21 = 15 + 6$;

11(a). $C(8,4)$;

11(b). $C(8,4)/2$;

11(c). $2^8 - 2$;

12. $C(6,3) + C(6,2) + C(6,1) + C(6,0)$;

13(a). $C(10,5)$;

13(b). $2^{10} - 2$;

14. $C(21,5)C(5,3)8! + C(21,4)C(5,4)8! + C(21,3)C(5,5)8!$;

15. Calculate the sum of those odd numbers with distinct digits with no 0's, a 0 in the tens place, or a 0 in the hundreds place. No 0's: 5 choices for the ones place, then $8 \cdot 7 \cdot 6$ choices for the other three places; 0 in the tens place: 5 choices for the ones place and 1 choice for the tens place, then $8 \cdot 7$ choices for the other two places; 0 in the hundreds place: 5 choices for the ones place and 1 choice for the hundreds place, then $8 \cdot 7$ choices for the other two places;
$(5 \cdot 8 \cdot 7 \cdot 6) + (5 \cdot 1 \cdot 8 \cdot 7) + (5 \cdot 1 \cdot 8 \cdot 7) = 2240$;

16(a). $C(7,3) \cdot C(4,2)$;

16(b). $C(7,1) \cdot C(4,1) + C(7,2) \cdot C(4,2) + C(7,3) \cdot C(4,3) + C(7,4) \cdot C(4,4)$;

16(c). $C(10,3)$;

16(d). $C(7,2) \cdot C(4,2) - C(6,1) \cdot C(3,1)$;

17(a)(i). $C(9,4)$;

17(a)(ii). 6;

17(a)(iii). 2;

17(a)(iv). 1;

17(b). JAVA, JAVA, JAVA, JAVA, JAVA, C++, C++, C++, C++;

17(c)(i). $9!$;

17(c)(ii). $6 \cdot 4! \cdot 5!$;

17(c)(iii). $2 \cdot 4! \cdot 5!$;

17(c)(iv). $5 \cdot 4 \cdot 4 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 1 \cdot 1$;

18(a). $[C(3,1)C(27,2) + C(3,2)C(27,1) + C(3,3)C(27,0)] \times [C(12,1)C(138,2) + C(12,2)C(138,1) + C(12,3)C(138,0)]$;

18(b). $C(30,3) \cdot C(150,3) - C(27,3) \cdot C(138,3)$;

19(a). $\binom{n}{m}\binom{m}{k} = \frac{n!}{m!(n-m)!}\frac{m!}{k!(m-k)!} = \frac{n!}{(n-m)!k!(m-k)!}$;
$\binom{n}{k}\binom{n-k}{m-k} = \frac{n!}{k!(n-k)!}\frac{(n-k)!}{(m-k)!((n-k)-(m-k))!} = \frac{n!}{k!(m-k)!(n-m)!}$;

20. Picking r items from n is the same as not picking $n-r$ items from n.

21. Sum the entries in the row labeled n, i.e., the $(n+1)^{st}$ row since the labels start with 0. For $n = 2$: $1+2+1 = 4$; For $n = 3$: $1+3+3+1 = 8$; For $n = 4$: $1+4+6+4+1 = 16$; in general, 2^n.

22. use repeated applications of Theorem 2.2 and the fact that $\binom{n}{0} = \binom{n+1}{0}$;

23. A set consists of n male items and m female items for a total of $n+m$ items. Left side: number of ways to pick a subset of size r from this set of $n+m$ items. Right side: Picking a subset of size r can be done by picking 0 males & r females, or 1 male and $r-1$ females, or ..., or r males & 0 females.

24(a). $\left\langle {n \atop r} \right\rangle = \binom{n+r-1}{r}$ and $\left\langle {n \atop r-1} \right\rangle + \left\langle {n-1 \atop r} \right\rangle = \binom{n+r-2}{r-1} + \binom{n+r-2}{r}$ are equal using equation (2.3);

25.

$$\left\langle {n \atop r} \right\rangle = \binom{n+r-1}{r} = \frac{(n+r-1)!}{r!(n-1)!}$$

and

$$\frac{n}{r}\left\langle {n+1 \atop r-1} \right\rangle = \frac{n}{r}\binom{n+r-1}{r-1} = \frac{n}{r}\frac{(n+r-1)!}{(r-1)!n!} = \frac{(n+r-1)!}{r!(n-1)!}$$

and

$$\frac{n+r-1}{r}\left\langle {n \atop r-1} \right\rangle = \frac{n+r-1}{r}\binom{n+r-2}{r-1} = \frac{n+r-1}{r}\frac{(n+r-2)!}{(r-1)!(n-1)!} = \frac{(n+r-1)!}{r!(n-1)!};$$

26(a). In the sequence of n 1's, t could be any integer from 0 to n, inclusive;

26(b). The sequence is strictly increasing when $\binom{n}{i} < \binom{n}{i+1}$ or $\frac{n!}{i!(n-i)!} < \frac{n!}{(i+1)!(n-i-1)!}$ or $i!(n-i)! > (i+1)!(n-i-1)!$ or $n-i > i+1$ or $i < \frac{n-1}{2}$. Similarly, it is strictly decreasing when $i > \frac{n-1}{2}$.

26(c). By 26(b), the largest entry in the sequence occurs when $i = \lfloor \frac{n}{2} \rfloor$. That is, at $\binom{n}{\lfloor n/2 \rfloor}$;

Section 2.8.

1(a). No;

1(b). No;

1(c). Yes;

1(d). No;

1(e). No;

2(a). 1/2;

2(b). 1/36;

2(c). 1/3;

3(a). $\frac{3}{8}$;

3(b). $\frac{1}{2}$;

3(c). $\frac{3}{4}$;

4. $\dfrac{C(3,2)\cdot 2 + C(3,3)}{3^3} = \dfrac{7}{27}$;

5. $\frac{15}{16}$;

6. $\frac{16}{4^5}$;

7. $\frac{2}{13}$;

8. $\frac{4}{52}\cdot\frac{4}{52}$;

9. $\frac{5}{16}$;

10. $\frac{5}{8}$;

11. $\dfrac{C(4,3) + C(4,4)}{4^2} = \dfrac{5}{16}$;

12. $\frac{1}{4}$;

13. $\frac{21}{32}$;

14. $\frac{\frac{20\cdot 18\cdot 16\cdot 14}{4!}}{C(20,4)}$;

15(a). $C(6,2)/2^6 + C(6,3)/2^6$.

15(b). $C(6,2)/2^6 + C(6,4)/2^6$.

15(c). $C(6,2)/2^6 + 2^5/2^6 - C(5,1)/2^6$.

15(d). $C(6,0)/2^6 + C(6,2)/2^6 + C(6,4)/2^6 + C(6,6)/2^6$.

15(e). $C(5,1)/2^6 + C(5,3)/2^6 + C(5,5)/2^6$.

16(a). probability of $E = \frac{n(E)}{n(S)}$ and probability of $E^c = \frac{n(S)-n(E)}{n(S)} = 1 - \frac{n(E)}{n(S)}$.

16(b). E and F disjoint implies $n(E\cup F) = n(E) + n(F)$.
So, probability of $E\cup F = \frac{n(E\cup F)}{n(S)} =$
$\frac{n(E)+n(F)}{n(S)}$ and probability of $E +$ probability of $F = \frac{n(E)}{n(S)} + \frac{n(F)}{n(S)}$.

16(c). E and F not disjoint implies $n(E \cup F) = n(E) + n(F) - n(E \cap F)$.
So, probability of $E \cup F = \frac{n(E \cup F)}{n(S)} = \frac{n(E)+n(F)-n(E \cap F)}{n(S)}$ and probability of $E +$ probability of $F -$ probability of $E \cap F = \frac{n(E)}{n(S)} + \frac{n(F)}{n(S)} - \frac{n(E \cap F)}{n(S)}$.

17(a). $2^2/2^4 + 2^2/2^4 - 1/2^4$.

17(b). $2^3/2^4 + 2/2^4 - 1/2^4$.

Section 2.9.

1(a). *aaaa*, *aaab*, *aaba*, *abaa*, *baaa*, *aabb*, *abab*, *abba*, *baab*, *baba*, *bbaa*, *abbb*, *babb*, *bbab*, *bbba*, *bbbb*;

1(b). $aa, ab, ac, ba, bb, bc, ca, cb, cc$;

1(c). $\{a, a, a, a\}, \{a, a, a, b\}, \{a, a, b, b\}, \{a, b, b, b\}, \{b, b, b, b\}$;

1(d). $\{a, a\}, \{a, b\}, \{a, c\}, \{b, b\}, \{b, c\}, \{c, c\}$;

2(a). $P^R(2, 4) = 2^4$;

2(b). $P^R(3, 2) = 3^2$;

2(c). $C^R(2, 4) = C(2 + 4 - 1, 4) = 5$;

2(d). $C^R(3, 2) = C(3 + 2 - 1, 2) = 6$;

3(a). $P^R(3, 7) = 3^7$;

3(b). $C^R(4, 7) = C(4 + 7 - 1, 7)$;

4. $C^R(4, 8) = C(4 + 8 - 1, 8)$;

5. $C^R(5, 12) = C(5 + 12 - 1, 12) = 1820$;

6. $P^R(5, 8) - P^R(5, 7) = 5^8 - 5^7$;

7(a). $C^R(4, 2) = C(4 + 2 - 1, 2) = 10$;

7(b). 1560;

8(a). $C^R(3, 82) = C(3 + 82 - 1, 82) = 3486$;

8(b). $C^R(2, 82) = C(2 + 82 - 1, 82) = 83$;

9. $P^R(4, 12) = 4^{12}$;

10. $\sum_{j=0}^{400} C^R(3, j)$;

11. $\sum_{j=0}^{nl} C^R(m, j)$.

Section 2.10.

1(a).

Distribution

		1	2	3	4	5	6	7	8
Cell	1	abc		ab	c	ac	b	bc	a
	2		abc	c	ab	b	ac	a	bc

1(d).

Distribution

		1	2	3	4
Cell	1	aaa		aa	a
	2		aaa	a	aa

3(b). $2^4 = 16$;

3(c). $4^2 = 16$;

3(e). $C(2+4-1,4) = 5$;

3(f). $C(4+2-1,2) = 10$;

4(a). $S(3,1) + S(3,2) = 4$;

4(d). Number of partitions of 3 into two or fewer parts = 2;

5(a). $2!S(3,2) = 6$;

5(d). $C(2,1) = 2$;

6(a). $S(3,2) = 3$;

6(d). Number of partitions of 3 into exactly two parts = 1;

7(a). $\{1,1,1,1\}, \{1,1,2\}, \{2,2\}, \{4\}, \{1,3\}$;

9(a). 1;

9(b). 1;

9(c). 1;

9(d). $\binom{n}{2}$;

9(e). 1;

10. 12 indistinguishable balls (misprints), 5 distinguishable cells (kinds of misprints), cells can be empty: $C(5+12-1,12)$;

11. 25 indistinguishable balls (misprints), 75 distinguishable cells (pages), cells can be empty: $C(75+25-1,25)$;

12. 30 distinguishable balls (errors), 100 indistinguishable cells (codewords), cells can be empty: $S(30,1) + S(30,2) + S(30,3) + \cdots + S(30,100)$;

13. 9 distinguishable balls (passengers), 5 indistinguishable cells (floors), cells can be empty: $S(9,1) + S(9,2) + S(9,3) + S(9,4) + S(9,5)$;

14. 10 indistinguishable balls (lasers), 5 indistinguishable cells (tumors), cells can be empty: Number of partitions of 10 into 5 or fewer pieces;

15. $C(6 + 30 - 1, 30)$;

16. 10 balls (customers), 7 cells (salesmen), cells can't be empty: answer depends on the choice (interpretation) of distiniguishable balls and/or cells;

17. $\dfrac{10!}{2^5 5!} = 945$.

18. $4!S(6,4)$ assuming both jobs and workers are distinguishable;

20. $1/C(11,8)$;

22(a). $S(n,k)$ counts the number of ways to place n distinguishable balls into k indistinguishable cells with no cell empty. Consider the n^{th} ball. Either it is by itself in a cell OR it is with other balls. $S(n-1,k-1)$ counts the number of ways where the n^{th} ball is by itself: To assure it is by itself, place the remaining $n-1$ balls in $k-1$ cells with no cell empty and then put the n^{th} ball in its own k^{th} cell. $kS(n-1,k)$ counts the number of ways where the n^{th} ball is with other balls in some cell: To assure that it is with other balls, place the remaining $n-1$ balls in k cells *with no cell empty* and then put the n^{th} ball in one of the k (nonempty) cells (i.e., there are k choices for the n^{th} ball);

22(c). 90;

24(a). 16. One ordering of $\{1,1,1,1,1\}$, 4 orderings of $\{1,1,1,2\}$, 3 orderings of $\{1,2,2\}$, 3 orderings of $\{1,1,3\}$, 2 orderings of $\{2,3\}$, 2 orderings of $\{1,4\}$, and 1 ordering of $\{5\}$;

24(b). 4. $\{2,3\}$, $\{3,2\}$, $\{1,4\}$, $\{4,1\}$;

24(c). The number of partitions of n into exactly k parts is Case 4b in Table 2.9, page 54. Saying that "order matters" is the same as saying that the cells are now distinguished. Thus, we are now in Case 2b in Table 2.9, page 54 which is counted by $C(n-1,k-1)$.

Section 2.11.

1(a). 630;

1(d). 90/729;

2. $C(n;1,1,1,\ldots,1) = \binom{n}{1,1,1,\ldots,1} = \dfrac{n!}{1!1!1!\cdots 1!} = n!$;

6(a). $C(7;4,1,2) = 105$;

9(a). $C(9;3,2,1,1,1,1) = 30,240$;

9(b). $\dfrac{C(9;3,2,1,1,1,1)}{26^9} = \dfrac{30,240}{26^9} \approx 5.56957 \times 10^{-9}$;

10. $4C(10;4,2,2,2) + C(4,2)C(10;3,3,2,2)$;

11. 4!. Since there are 5 a's and 4 non-a's, any such permutation must start with an a and alternate a's and non-a's. Thus, we need only count how to order the four non-a's;

13(a). $C(5;3,1,1)$;

16(b). $P(4;3,1)$;

16(d). $P(4;2,1,1) = 12$.

***Section 2.12*.**

1(a). $3! = 6$;

1(b). $\dfrac{5!}{2} = 60$;

1(c). CAAGCUGGUC;

2. $C(10;2,3,3,2)$;

3(a). $\dfrac{4!}{3!} = 4$;

3(b). 4!;

3(c). GUCGGGUU and GGGUCGUU;

7(a). $\dfrac{3!}{2!} = 3$;

7(b). $\dfrac{5!}{3!2!} = 10$;

7(c). 00101010, 01001010 and 01010010;

8. GCGUGU and GUGCGU;

10. yes: 00101010 and 01001010 have the same breakup.

***Section 2.13*.**

1. $9/C(12,4)$;

3(a). The given *order* has a probability of $\frac{1}{C(13;8,5)}$ of being observed;

3(b). $\frac{C(9;8,1)}{C(13;8,5)}$. Group the 5 sick trees together as one unit S^*. Then the number of orders of 1 S^* and 8 W's is $C(9;8,1)$;

5(a). Group the 6 infested houses together as one unit I^*. Then the number of orders of 1 I^* and 5 noninfested houses is $C(6;5,1)$;

5(b). Only 1: start with an infested and alternate;

8(a). $7/4^4$;

9. $C(17;13,4)$;

10. $\binom{15}{5,4,5,1}$. This is equivalent to RNA chains of length 15 having five A's, four U's, five H's and one G, where H = "CG."

Section 2.14.

1(a). $\binom{5}{0}x^5 + \binom{5}{1}x^4y^1 + \binom{5}{2}x^3y^2 + \binom{5}{3}x^2y^3 + \binom{5}{4}x^1y^4 + \binom{5}{5}y^5$;

1(b). $\binom{3}{0}a^3 + \binom{3}{1}a^2 2b + \binom{3}{2}a(2b)^2 + \binom{3}{3}(2b)^3 = a^3 + 6a^2b + 12ab^2 + 8b^3$;

1(c). $\binom{4}{0}(2u)^4 3v^0 + \binom{4}{1}(2u)^3 3v^1 + \binom{4}{2}(2u)^2 3v^2 + \binom{4}{3}(2u)^1 3v^3 + \binom{4}{4}(2u)^0 3v^4 = 16u^4 + 96u^3v + 216u^2v^2 + 216u^1v^3 + 81v^4$;

2(a). $4368 = \binom{16}{11}1^5$;

2(b). $2912 = \binom{14}{11}2^3$;

2(c). $2^{11} = \binom{11}{0}2^{11}$;

3. $C(12,9)C(4,0) + C(12,8)C(4,1) + \cdots + C(12,5)C(4,4)$;

4. $C(10,8)C(6,0) + C(10,7)C(6,1) + \cdots + C(10,2)C(6,6)$;

5. $\displaystyle\sum_{\substack{i,j,k\geq 0 \\ i+j+k=n}} \binom{n}{i,j,k} a^i b^j c^k$;

6. $\binom{6}{2,2,2} = 90$;

7. $\binom{5}{1,1,3} = 20$;

8. $\displaystyle\sum_{\substack{n_i\geq 0,\forall i \\ n_1+n_2+\cdots+n_k=n}} \binom{n}{n_1,n_2,\ldots,n_k} a_1^{n_1}a_2^{n_2}\cdots a_k^{n_k}$;

9. $\binom{6}{3,1,2} = 60$;

10. $\binom{6}{1,2,2,1} \cdot 2 = 360$;

11. $\binom{12}{3,2,1,6} \cdot 5^2 \cdot 2 \cdot 2^6 = 177,408,000$;

13. $\frac{1}{2} \cdot 2^{12} = 2048$;

15. $\frac{1}{2} \cdot 2^n = 2^{n-1}$;

16(a). 3^n;

16(b). 5^n;

16(c). $(1+x)^n$;

16(d). $n(n-1)2^{n-2}$;

16(e). $n2^{n-1}$;

17(a). Start with $(x+2)^n = \sum_{k=0}^{n} \binom{n}{k} 2^{n-k} x^k$, differentiate. and then let $x = 2$;

17(b). Start with $(x+2)^n = \sum_{k=0}^{n} \binom{n}{k} 2^{n-k} x^k$, differentiate. and then let $x = 1$.

Section 2.15.

1(a). $\{1,2\}, \{1,3\}, \{1,2,3\}$;

2(a). $\{1,2,5\}, \{1,3\}, \{2,3\}, \{1,4\}, \{2,4\}, \{3,4\}, \{4,5\}$;

3(b). $\frac{1}{5}$ for each;

6(a). yes - $[5; 5, 2, 1]$;

11(b). nonpermanent: $\frac{9!9!}{3!16!}$.

Section 2.16.

1(a). 2143 precedes 3412;

2(b). 0101;

3(c). $\{1,3,5,6\}$;

4(c). 152634;

11. $\{1,2\}, \{1,3\}, \{1,4\}, \{1,5\}, \{2,3\}, \{2,4\}, \{2,5\}, \{3,4\}, \{3,5\}, \{4,5\}$;

12. $0111 \cdots 1$;

21(b). 4.

Section 2.17.

3. 456123;

5(c). $n12 \cdots (n-1)$;

5(d). 2;

7(b). 5: 15423 → 14523 → 14253 → 14235 → 12435 → 12345;

9(a). 2: first, transpose 7 and 46 to get 5123467; then transpose 5 and 1234;

10(a). 2.

Section 2.18.

1(b). yes;

1(d). yes;

4. $f(n) \leq k_1 g(n)$ for some positive constant k_1 and $n \geq r_1$; $g(n) \leq k_2 h(n)$ for some positive constant k_2 and $n \geq r_2$; then $f(n) \leq k_1 k_2 h(n)$ for positive constant $k_1 k_2$ and $n \geq \max\{r_1, r_2\}$;

8(a). yes;

9(a). yes;

9(f). yes;

9(i). yes;

12. The third is "big oh" of the other two; the first is "big oh" of the second.

Section 2.19.

1(a). 27;

2(a). 53;

3. 6;

4(a). 4;

6. Yes; use Corollary 2.15.1 and the fact that the average number of seats per car is $\frac{465}{95} \approx 4.89$;

9(a). 9;

9(b). 20;

13(a). longest increasing is 6, 7 or 5, 7 and longest decreasing is 6, 5, 4, 1;

14. $13, 14, 15, 16, 9, 10, 11, 12, 5, 6, 7, 8, 1, 2, 3, 4$;

15. let the pigeons be the 81 hours and the five holes be days 1 and 2, 3 and 4, 5 and 6, 7 and 8, and 9 and 10;

24. if there are n people, each has at most $n - 1$ acquaintances;

29(a). $X = \{\{a, b\}, \{b, c\}, \{c, d\}\}, Y = \{\{a, c\}, \{b, d\}, \{a, d\}\}$;

30(a). 2.

Additional Exercises for Chapter 2.

No answers provided.