Applied Combinatorics

by Fred S. Roberts and Barry Tesman

Answers to Selected Exercises[1]

Chapter 3

Section 3.1.

1(a). $V = \{$Chicago (C), Springfield (S), Albany (A), New York (N), Miami (M)$\}$;

1(b). $A =$
$\{$(C, S), (S, C), (C, N), (N, C), (A, N), (N, A), (C, M), (M, C), (N, M), (M, N)$\}$;

4(b). In $G_3, E = \{\{u, v\}, \{v, w\}, \{u, w\}, \{x, y\}, \{x, z\}, \{y, z\}\}$;

17(a). yes;

17(b). no;

19. 15;

20. 32;

23. no;

24. yes.

Section 3.2.

1(d). yes;

5. D_4: no; D_6: yes;

6(a). no;

6(e). yes;

9(b). D_4: yes; D_8: no;

10(c). D_4: yes; D_8: yes;

13. D_8: $\{p, q, r, s, t\}, \{u\}, \{v\}, \{w\}$;

18. *Hint:* Use induction on the number of vertices;

[1]More solutions to come. Comments/Corrections would be appreciated and should be sent to: Barry Tesman (tesman@dickinson.edu) or Fred Roberts (froberts@dimacs.rutgers.edu).

25(a). yes;

30(a). 9;

30(b). $2\binom{n-1}{2} + (n-1) = (n-1)^2$.

***Section 3.3*.**

3(a). (b): yes;

3(b). (b): 3;

5. no;

8. 4;

18. no;

21. (b):yes;

41(a). 4;

54(b). Z_n, n odd, $n \geq 3$;

55(a). $\omega(G) = \alpha(G^c)$;

54(b). Z_5 plus a vertex adjacent to two consecutive vertices of Z_5.

***Section 3.4*.**

1(a). $x(x-1)^3$;

2(a). 24;

2(c). 48;

6. $[P(I_2, x) - P(I_1, x)][P(I_1, x) - 2]^2$;

9(a). $5 \cdot 4!$; $\binom{5}{2} \cdot 4!$;

11(a). 2;

11(c). (a):0;

13(d). $P(x) \neq x^n$ and the sum of the coefficients is not zero;

20(b). yes;

25(c). $(-1)^{n-1}(n-1)!$.

***Section 3.5*.**

4(a). 11;

4(b). 9;

9. $n - k$;

13. 16;

16. There are too few edges to have a spanning tree; alternatively, the deleted edge was the only simple chain between its end vertices;

19. 2 if $n \geq 2$, since $2(2-1)^{n-1} > 0$ and $1(1-1)^{n-1} = 0$; 1 if $n = 1$;

26. *Hint:* The sum of the degrees is $4k + m$ and we have a tree;

29(b). yes;

31(a). 6;

32(b). $\binom{8}{2}6! = 20,160$.

Section 3.6.

2(a). a: 0; b, c: 1; d, e, f, g: 2; h, i, j, k: 3;

3(a). 3;

4(a). $\{d, e, h, i\}$;

7(a). The children of vertex 1 are 2 and 3 and the children of vertex 2 are 4 and 5;

11(a). 6;

21. $[1 \cdot 2^0 + 2 \cdot 2^1 + 3 \cdot 2^2 + \cdots + (h+1)2^h]/n$, where h is the height;

26. 3 4 1 2, 3 1 4 2, 3 1 2 4, 1 3 2 4, 1 2 3 4, 1 2 3 4;

31. 10;

43(b). 240.

Section 3.7.

1(a). For D_1 :
$$\begin{array}{c} \\ u \\ v \\ w \\ x \end{array}\begin{array}{c} \begin{array}{cccc} u & v & w & x \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix} \end{array};$$

2(h).
$$\begin{array}{c|ccccc} & \{a,b\} & \{b,c\} & \{a,d\} & \{b,e\} & \{d,e\} \\ \hline a & 1 & 0 & 1 & 0 & 0 \\ b & 1 & 1 & 0 & 1 & 0 \\ c & 0 & 1 & 0 & 0 & 0 \\ d & 0 & 0 & 1 & 0 & 1 \\ e & 0 & 0 & 0 & 1 & 1 \end{array};$$

7. For G_1 : $\begin{matrix} & u & v & w \\ u & 0 & 1 & 0 \\ v & 1 & 0 & 1 \\ w & 0 & 1 & 0 \end{matrix}$; **13**. For D_4 : $\begin{matrix} & u & v & w & x & y & z \\ u & 1 & 1 & 1 & 1 & 1 & 1 \\ v & 1 & 1 & 1 & 1 & 1 & 1 \\ w & 1 & 1 & 1 & 1 & 1 & 1 \\ x & 0 & 0 & 0 & 1 & 1 & 1 \\ y & 0 & 0 & 0 & 1 & 1 & 1 \\ z & 0 & 0 & 0 & 1 & 1 & 1 \end{matrix}$;

21(a). There is a path from i to j if and only if there is a path of length at most $n-1$;

24(a). j is in the strong component containing i if and only if $r_{ij} = 1$ and $r_{ji} = 1$;

30. it gives the number of vertices that edges i and j have in common;

33. yes: take Z_4 as in Figure 3.22 and append x adjacent to a and b and y adjacent to b and c; repeat with Z_4 and x as above, but take y adjacent to c and d; relabel edges.

Section 3.8.

4. 7;

5(a). $\{a, c, e\}$;

5(f). $\{a, b, d\}$;

8(a). Let 4 "red edges from one vertex" be $\{a, b\}$, $\{a, c\}$, $\{a, d\}$, $\{a, e\}$. If any one of the edges $\{b, c\}$, $\{b, d\}$, $\{b, e\}$, $\{c, d\}$, $\{c, e\}$, $\{d, e\}$ is red then there will be 3 vertices all joined by red edges. If they are all blue then vertices b, c, d, e are all joined by blue edges;

9(a). Yes;

11(c). 4;