Applied Combinatorics

by Fred S. Roberts and Barry Tesman

Answers to Selected Exercises[1]

Chapter 4

Section 4.1.

2. Consider a brother and a sister;

4. Less than;

6(a). It is a complete graph with loops at every vertex;

8(b). Suppose that R^c is not symmetric. Then, for some $a, b \in X$, aR^cb but $\sim bR^ca$. Therefore, R is not symmetric since bRa but $\sim aRb$;

10. The relation $(X, R \cap S)$ is reflexive, symmetric, asymmetric, antisymmetric, and transitive;

16. Consider the binary relation (X, R) and suppose that aRa for some $a \in X$. By asymmetry, it must be the case that $\sim aRa$, which is a contradiction;

19(a). $X = \{a, b, c\}$ and $R = \{(a, c)\}$;

20. (a), (d), (e), and (g) are equivalence relations;

25. Reflexive and Symmetric hold;

26(e)i. aSb iff $\sim aRb$ & $\sim bRa$ iff $a \not> b$ & $b \not> a$ iff $a = b$;

27. 2^{n^2}.

Section 4.2.

1(b). Yes.

2(b). Yes.

3(b). No.

4(b). No.

5(b). No.

[1] More solutions to come. Comments/Corrections would be appreciated and should be sent to: Barry Tesman (tesman@dickinson.edu) or Fred Roberts (froberts@dimacs.rutgers.edu).

6(b). No.

7(b). No.

8(b). No.

9(b). No.

10(b). No.

13(c). $K = \{(1,3),(2,3),(3,4)\}$.

19(a). $L_{S^{-1}} = [x_n, x_{n-1}, \ldots, x_1]$.

19(b). $L_S \cap L_{S^{-1}} = \emptyset$.

21. Transitive and complete, but not asymmetric: $X = \{a\}$ and $R = \{(a,a)\}$. Transitive and asymmetric, but not complete: $X = \{a,b,c\}$ and $R = \{(a,b),(a,c)\}$. Complete and asymmetric, but not transitive: $X = \{a,b,c\}$ and $R = \{(a,b),(b,c),(c,a)\}$.

25(a). Yes.

30(a). No.

33(a). w_1 and m_2 are both better off by leaving their assigned partners and marrying each other.

Section 4.3.

1. No.

4(a). 3.

4(e). 4.

5(c). 2.

9(a). $[\hat{1}, x, y, d, z, a, b, c, \hat{0}]$.

12. Strict partial order (c) of Figure 4.23 has dimension 2.

17. $[a,x]$, $[b,y]$, $[c,z]$, $[d,w]$.

18. $\{u\}$, $\{y,w\}$, $\{z,v\}$, $\{x\}$.

Section 4.4.

1(a). No for both strict partial orders

1(b). No for both strict partial orders

2(b). (a): $\hat{0}$; (b): d; (c): $\hat{0}$.

3(a). (a): Not a lattice; (b): Not a lattice.

11.

a	b	c	$a \wedge (b \vee c)$	$(a \wedge b) \vee (a \wedge c)$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
1	0	0	0	0
0	1	1	0	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

12. (a): No.

16(c).

p	q	q'	$p \wedge q'$	$(p \wedge q') \to q$
F	F	T	F	T
F	T	F	F	T
T	F	T	T	F
T	T	F	F	T

17(b). $p =$ Pete loves Christine; $q =$ Christine loves Pete; $p \wedge q =$ Pete and Christine love each other.

p	q	$p \wedge q$
F	F	F
F	T	F
T	F	F
T	T	T

18(b). Equivalent.

20(a).

x_1	x_2	$x_1' \wedge x_2$	$x_1 \vee x_2$	$(x_1' \wedge x_2) \vee (x_1 \vee x_2)$
1	1	0	1	1
1	0	0	1	1
0	1	1	1	1
0	0	0	0	0