Applied Combinatorics

by Fred S. Roberts and Barry Tesman

Answers to Selected Exercises[1]

Chapter 9

Section 9.1.

1.

1	4	3	2
1	4	3	2
1	4	3	2
1	4	3	2

;

2(b). it must be a multiple of 6;

6(a). 21.

Section 9.2.

1(a). no;

3(a). no;

4(a). yes;

5(a). cannot be sure;

5(c). cannot be sure;

8. yes;

9. no;

11(a). no;

11(b). it is at most 7.

17. $A^{(1)} = \begin{array}{|ccc|}\hline 3 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \\ \hline\end{array}$, $A^{(2)} = \begin{array}{|ccc|}\hline 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \\ \hline\end{array}$;

[1] More solutions to come. Comments/Corrections would be appreciated and should be sent to: Barry Tesman (tesman@dickinson.edu) or Fred Roberts (froberts@dimacs.rutgers.edu).

19. $\begin{bmatrix} 1 & 1 & 1 & 4 \\ 1 & 2 & 2 & 1 \\ 1 & 3 & 3 & 2 \\ 1 & 4 & 4 & 3 \\ 2 & 1 & 2 & 3 \\ 2 & 2 & 1 & 2 \\ 2 & 3 & 4 & 1 \\ 2 & 4 & 3 & 4 \\ 3 & 1 & 3 & 1 \\ 3 & 2 & 4 & 4 \\ 3 & 3 & 1 & 3 \\ 3 & 4 & 2 & 2 \\ 4 & 1 & 4 & 2 \\ 4 & 2 & 3 & 3 \\ 4 & 3 & 2 & 4 \\ 4 & 4 & 1 & 1 \end{bmatrix}$;

22(a). yes;

22(b). no;

22(c). yes.

Section 9.3.

1(a). 2;

2(b). $a + b = 9, a \times b = 8$;

5(c). no;

7(b). 258;

10(a).

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

×	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

11(b). 2;

13(a). 3; 7;

15(a). no.

Section 9.4.

1(a). not a BIBD;

2(a). $b = 50, r = 25$;

3(a). $r(k-1) \neq \lambda(v-1)$;

7. no: $b \geq v$ fails;

9(a). $\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccccc} \{1,2\} & \{1,3\} & \{2,4\} & \{1,2,3\} & \{2,3,4\} \end{array} \\ \begin{pmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix} \end{array}$;

10(a). $\begin{bmatrix} 3 & 2 & 2 & 2 \\ 2 & 3 & 2 & 2 \\ 2 & 2 & 3 & 2 \\ 2 & 2 & 2 & 3 \end{bmatrix}$;

18. 737;

21. $b = 26, v = 13, r = 20, k = 10, \lambda = 15$;

26(a). this is a $(4m-1, 2m-1, m-1)$-design, $m = 2^3$;

35(a). no: $k - \lambda$ is not a square;

41. take two copies of each block of a (31, 15, 7)-design.

Section 9.5.

1(d). (P3);

2(a). There are 9 distinct points, no 3 of which lie on the same line;

4. 21;

8(a). no;

9(a). $v = 31, k = 6, \lambda = 1$;

10(a). yes (Corollary 9.27.1);

14(a). yes (but cannot be sure);

16(a). 1;

17(a). 1;

22(b). if we take
$U_3 = \{1,3,5,7\}, V_2 = \{2,3,4,13\}, W_{11} = \{3,6,8,11\}, W_{21} = \{3,9,10,12\}$, then the point 3 is associated with (3, 2) and (3, 2, 1, 1);

22(c). $a_{32}^{(1)} = 1, a_{32}^{(2)} = 1$;

23(a). (2, 3) is associated with (2, 3, 1, 2);

23(b). $W_{12} = \{(1,2),(2,1),(3,3)\}$;

23(e). W_{12} is now $\{(1,2),(2,1),(3,3),w_1\}$, the finite points are all (i,j) with $1 \le i,j \le 3$, and the infinite points are u,v,w_1,w_2;

23(f). m^2+m+1 lines, including the line at infinity.