LEADERSHIP PROGRAM IN DISCRETE MATHEMATICS

Instructor's Notes

Revised July 21, 1999

Workshop 10 — Generating Fractals

Materials Needed **Allocated Time**

Activity #1 — Review of Iteration ..... 20 minutes

Activity #2 — Reduce, Replicate, and Rebuild ..... 45 minutes

Activity #3 — Rotations and the Wallpaper Fractal ..... 30 minutes

- **Cut-out of TSP of bottom right of TSP #21**
- **Large poster sheets of white paper for posting fractals**

Activity #4 — Generating Personalized Fractals ..... 30 minutes

- **Each person should receive a quarter of a blank transparency.**

..... TOTAL WORKSHOP TIME: 125* minutes

* In addition, ten minutes are allocated for a break in this 2 ¼ hour workshop.

Activity #1 — Being Precise: Review of Iteration
(Allocated time = 20 minutes)

A. Start with half of the participants (the A's) in one room and the other half (the B's) in another room. Show the A's TSP #1 , while the B's look at TSP #2. Each person's task is to study the sequence of figures, and then write down an iterative rule that will generate this sequence of figures. Their rule must use words only, that is, no pictures. Explain also that after all of the participants have completed writing their

rules, the two groups will be combined, and each A will be paired with a B. A will study B's rule and attempt to generate the sequence of figures on TSP #2, while B will study A's rule and attempt to generate the sequence of figures in TSP #1 . Therefore, each person should be attempting to write a rule which is sufficiently explicit so that his or her partner will be able to reconstruct the figures on his or her transparency.

If only one room is available, this can be carried out in stages, with the A's looking at TSP #1 and writing their rules while the B's avert their eyes, and then afterwards the B's looking at TSP #2 and writing their rules while the A's avert their eyes. "No talking and no peeking!"

When both partners in a pair believe they have good rules, they should trade rules, and each partner should try to generate the sequence of figures (not just a single figure, but the first four or five in the sequence) based on the rule.

You can expect this to be harder than it sounds! Emphasize that there can be no talking, even if someone is in agony watching their partner attempt to draw the figures. The point of this lesson is the importance, and difficulty, of finding rigorous descriptions of these processes.

After a few minutes, review some of the rules that failed, letting the participants show how they failed and how they would fix them, and some of the rules that succeeded.

The figures involving the circles tend to be harder for the participants. Following the pattern in yesterday's workshop, they are likely to have rules of the form "whenever you see ... , do ... ." E.g., whenever you see a solid square, remove the upper right and lower left quadrants, or whenever you see an empty circle, put two circles inside such that etc. After completing Activity #2, return to these figures and reinterpret them in terms of "reduce, replicate and rebuild."

When all of these activities are over, review the figures that they have seen, and discuss the idea of self-similarity again, showing how an ideal figure can contain infinitely many reduced copies of itself. Note that square figure would not generate a fractal, because the end stage is just a line segment — a diagonal of the square. The circles figure would probably be considered a fractal, though some might object that it isn't infinitely complex. These examples nicely illustrate exactly where the subjectiveness of our definition of fractal appears.

Activity #2 — Reduce, Replicate and Rebuild.
(Allocated time = 45 minutes)

Before beginning this activity proper, mention to the participants that the theme of the day will be generating fractals. Our technique for doing so will be to Reduce, Replicate and Rebuild, as shown on TSP #3.

This transparency has room on the bottom for you to show examples, which are included on several of the later transparencies.

A. Place TSP #4 over TSP #3, and lead the participants through the generation of a fractal formed in a square by reducing by a factor of ½, and rebuilding by placing a copy in a single corner, say, the upper left, as prescribed on this TSP. Use blank transparencies overlaid on TSP #5 to carry out this activity and subsequent activities. (A second copy is provided in case you wish to draw directly on one of them.)

The resulting sequence of figures will "converge" to a single point at the corner of the square. Since this figure is not infinitely complex, it is not considered a fractal. A discussion of "convergence" will probably not be productive; informal statements like "the figures will tend toward a single point" or "the 'end stage' will be a single point at the corner of the square" will probably be sufficient. You can connect this to yesterday's activity where line segments in the triangle converged to a single point at the top.

It is not so bad at this point if they wonder why you are doing this. It will help to make the case where we place three copies (and where we get all the action) seem less arbitrary.

The little pictograms (to the right of "Iterating Instructions") on each of these TSPs serve two purposes: 1) to show graphically what the rule is in a single glance, and 2) to help you identify the slides quickly in case they get out of order!

B. Repeat the activity, but this time place two copies along the left side of the original square, as shown on TSP #7. In this case, as we iterate the procedure, each time taking two reduced copies and placing them in the two left squares, we converge to just a line segment along the left side of the square. Similarly, two copies placed along any other edge will result in a sequence of figures which converge to that edge of the square. These figures, also failing to be infinitely complex, are not fractal.

C. Repeat the activity (quicker now) placing two reduced copies in opposite corners, as shown on TSP #8. This time our figure will converge to a diagonal of the square.

The participants will recognize this as the same sequence of figures that they saw on TSP #2. At this point, go back to TSP #1 and TSP #2 and reinterpret the two sequences of figures in terms of Reduce, Replicate and Rebuild. It should surprise them, at least a little bit, that the circle figures can be so described; note that in this case, you have to add a circle in the rebuild step. Once again, of course, we don't have an infinitely complex figure, so it is not a fractal.

D. Before getting to part E, which discusses the really exciting case, consider the case where you place four copies, one in each of the four quadrants, as shown on TSP #9. After doing the other cases and not getting anything which is infinitely complex, and hence not a fractal, you can act as if you are hurrying up to get to the exciting part.

E. Finally, you can ask the participants if they think it's worth bothering to try the "three copies" case, as shown on TSP #10. They will probably insist that you should try this case, so do it on the overhead using the grids on TSP #5 through stages 0, 1 and 2; and when they see that something nontrivial seems to be happening, distribute Handout #1 with the square grids so that they can generate the figures.

Give them some time for this, and go around helping. Mistakes will abound. After a few minutes, review with them (by building on a TSP overlaid on TSP #5) the stage 3 figure, then have them work on stage 4. After a few more minutes, you can show TSP #11 which shows stages 1-5 of this figure. Gradually they will suspect that a Sierpinski-like structure is emerging. They are right!

TSP #12 shows stage 7 of this fractal. You should again talk about delicateness of the figure, its self-similarity (by covering up 2/3, 8/9, etc... of the figure) and any other features of the Sierpinski Triangle which you talked about yesterday.

There is quite a difference between the rules which generated the Sierpinski Triangle yesterday (remove the middle quarter of each shaded triangle) and the rule today which placed three copies of the reduce figure in three corners of a square. The next activity will help to make the connection.

F. After asking them if they are surprised that this very different (from yesterday's) rule generated such a similar figure, show stage 7 of the right Sierpinski-

like triangle, TSP #13. It will look a lot like the stage 7 above (with the squares) but made from triangles instead. Now show TSP #14, the right Sierpinski triangle stages 0-5, and help them to see the connection between this and both yesterday's equilateral Sierpinski triangle and today's squares.

The gap here is that yesterday, we generated the Sierpinski triangle by cutting out the middle quarters of the triangles, and today we generated a Sierpinski-like figure by reducing, replicating and rebuilding. These slides are designed to help the participants see the connection, and that the two methods generate precisely the same sequence of figures if we just let them act on right triangles instead of on a square and an equilateral triangle. TSP #14 is the "half-way" slide between these two rules.

Talk about how you can generate the triangle by reducing, replicating and rebuilding. Discuss how you have something like self-similarity — parts look essentially like the whole, but not exactly. You only have exact self-similarity when you consider the end product.

Emphasize that the important thing is the rule used to generate successive iterations of these figures, and not the shape that you start with.

Show TSP #15, TSP #16, and TSP #17 that use other shapes to start with but generate similar-looking figures. In case they ask, yes!, the mailboxes are in their resource books.

[Time for a 5-10 minute break]

Activity #3 — Introducing Rotations and the Wallpaper Fractal
(Allocated time = 30 minutes)

When we were making spirolaterals we saw how a small change in the sequence that generated the figure resulted in a major change in the end product. The same thing happens here. In the spirolaterals we had rotated copies of the initial figure. What if we introduce some minor change in our building algorithm for the Sierpinski triangle? Here we shall rotate some of our reduced replicas.

A. Show TSP #18 which gives a rule in the same format as the previous rules we've seen, but includes rotations in the description of the rebuilding step. Describe the building process through a few stages with them (using TSP #19 and TSP #20), so that they get the idea, but before you show them a stage, ask them if they can *visualize* what

the stage will look like.

Using an overlay on TSP #5, redo this process except starting with a square instead of a right triangle. Mention (again) that it is the building rule and not the starting point that really determines the structure of the resulting fractal. Build through stage 4 with them as seen on TSP #21. It is highly recommended that after you draw a reduced version in the lower left quadrant that you then trace the reduced version on an overlaid blank TSP, and then physically rotate it before drawing the rotated copies in the other two quadrants. This will help them see what you are doing, and will model a method that they will be encouraged to use during Activity #4 and during their study groups.

Note: In Activity #4 each table will be assigned their own particular generation rule so that they can generate their own fractal.

B. Leave TSP #21 on the overhead projector while Handout #2 = TSP #22 is being distributed. Hold up a copy of the handout and point out that each of these 9 figures is a copy of stage 6 of this fractal, starting with squares. To build a stage 7, we could skip the "Reduce" step and just replicate and rebuild. Taking 3 replicas of this stage 6 (cut from TSP #22) and rebuilding according to the same rule shown on TSP #21, demonstrate the construction on the overhead projector. Then indicate how you would build stage 8, and invite them to do this with their handouts. (Yes, a copy is provided in the resource book.)

As people complete their stage 8 versions, invite them to find 2 friends and make a stage 9. And then, if two groups who have made their stage 9's get together, they can complete a stage 10 (this takes 9 people.) Stage 11 takes 27 people, and we won't have enough to build a stage 12. Provide large poster-size sheets of white paper so that the participants have a "canvas" on which to build their creation. With 27+9+3+1+1/3 + 1/9 = 41 people, you can actually build an "evolutionary history" showing the progression from stage 6 to stage 11.

Activity # 4 — Generating Personalized Fractals
(Allocated time = 30 minutes)

A. The rule on TSP #18 shows just one way we could have placed three copies in the three non-blank quadrants. Ask how many different ways we could have chosen to rotate our reduced replicas before placing in the three designated quadrants, assuming that we placed them in the same three quadrants and left the remaining

quadrant blank (TSP# 23). There are 64 ways, as they should easily deduce by the multiplication rule. Then ask how many ways there would be if we fixed the lower left quadrant to be an unrotated copy. Now there are 16 ways, as shown on TSP #24. You can point out that rule "A" generates the Sierpinski Triangle, and that rule "H" is the rule we just followed in class.

Will these produce very different figures? Ask participants what they think, while Handout #3 (=TSP #25) is being distributed. This transparency shows 4 of the rules displayed on TSP #24, selected for their variety, and happening to rearrange to form the word "MINK." After a moment of discussion, perhaps explaining why we show three stages on this slide instead of two stages, (since squares alone don't display our rotations) assign each group one of the rules (or you can let them claim their own rules) so that each rule is assigned to at least 2 tables, if possible. Ask them to generate through stage 5 of their fractals, on HO #4, which won't take too long since we are using squares.

This tends to be tricky for the participants, because it involves a lot of mental geometry. This activity is done after the wallpaper fractal activity, so it is hoped that having the hands-on experience at this point will help them.

Give each participant a quarter-sheet of transparency film so that they can trace their reduced image prior to rotating it. This has helped some participants immensely in the past, especially if you demonstrated using it during Activity #3.

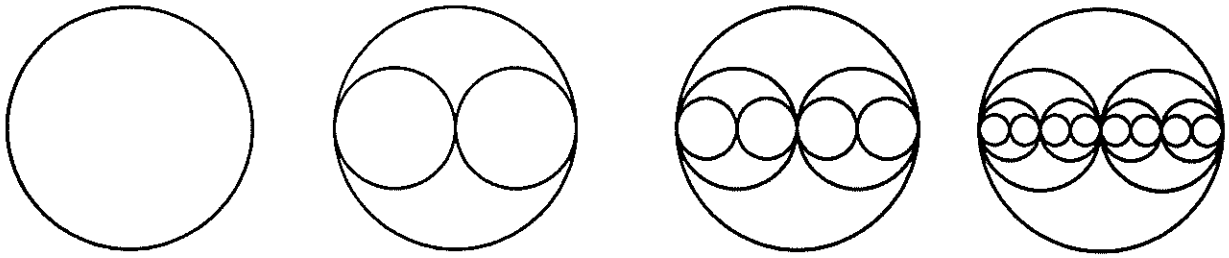
B. When all tables have created at least one good version of their very own fractal, put up TSP #27 showing stage 5 of each of these rules. Have the participants compare the four figures on the overhead with the four rules on their handout, and see if they can match the fractal with the rule. After two minutes or so, poll them for their answers by asking which figure was generated by rule M, and then which by rule I, etc... During the poll time, however, the groups which followed a particular rule are not allowed to vote. Only after the class has agreed on a particular answer should you then allow (force) the groups which generated that figure to explain why it makes sense that their rule should generate a figure which looks like that.

This is a place to encourage participant creativity as they vocalize discoveries about the various structures and discover similarities and differences among the figures. One of the points, again, is that if they are willing to experiment, they have many possibilities available to them for generating very interesting figures.

Write a few sentences (no drawings) which would enable your partner to draw the sequence of figures presented below.

Note that it may help to specify

1. A starting figure
2. A rule for getting from one stage to the next



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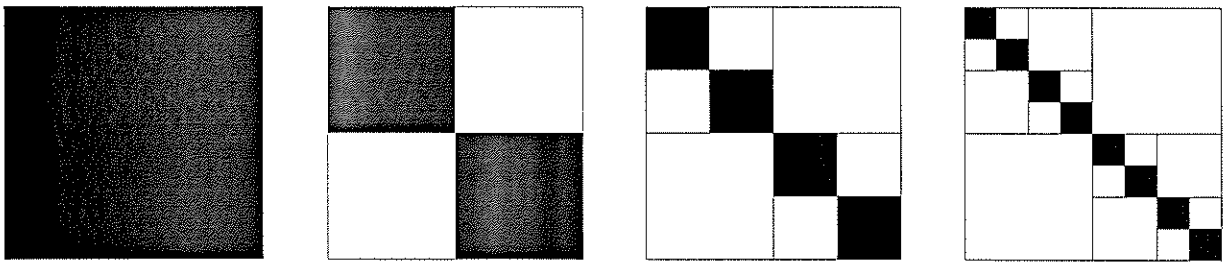


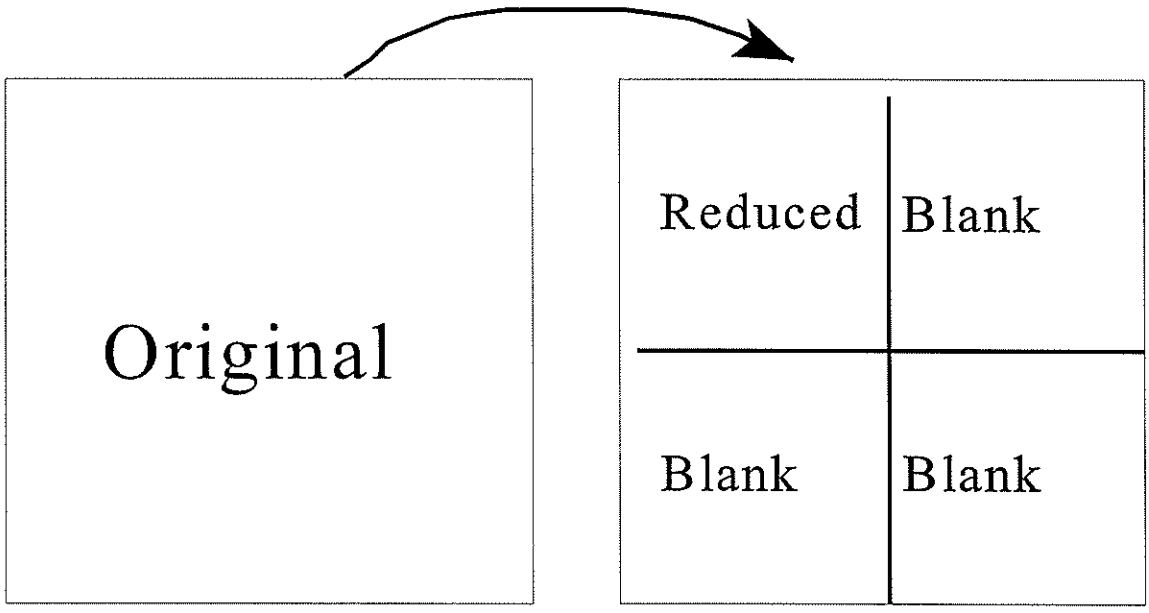
GENERATING FRACTALS

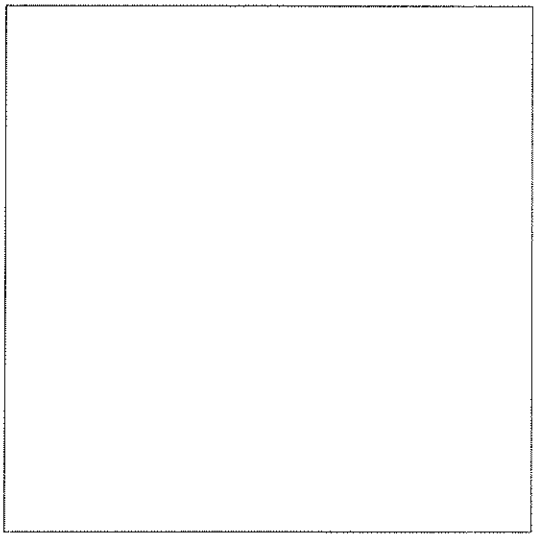
REDUCE: Reduce the current figure to some fraction (usually 1/2) of its original size

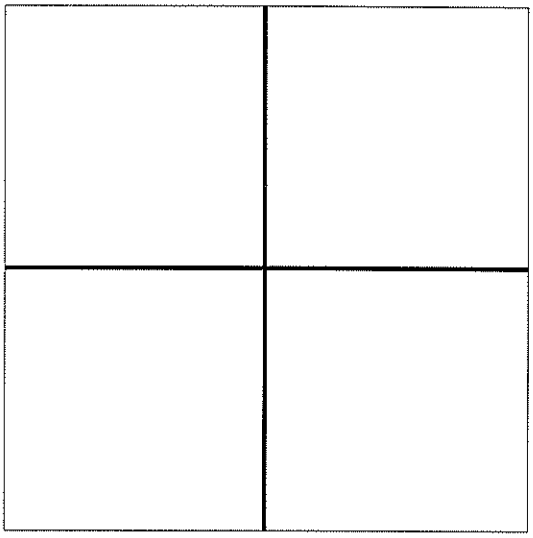
REPLICATE: Make several copies of the reduced figure

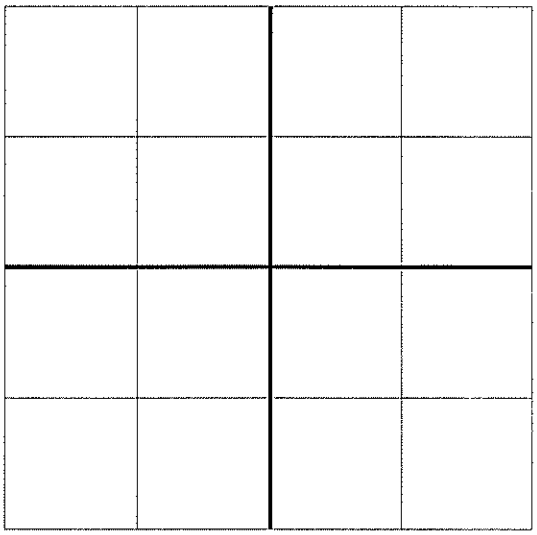
REBUILD: Put these copies together in a prescribed fashion

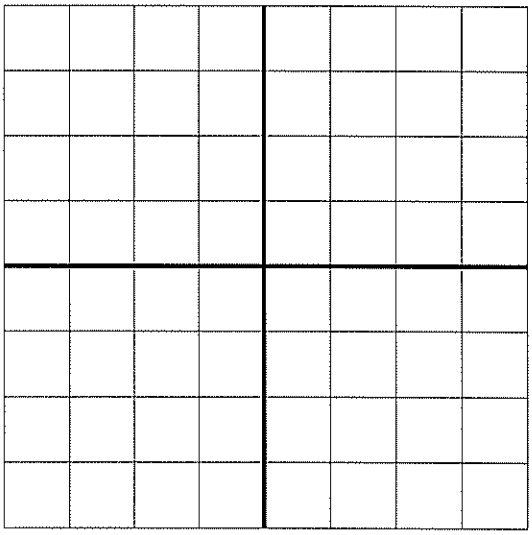
Iterating Instructions

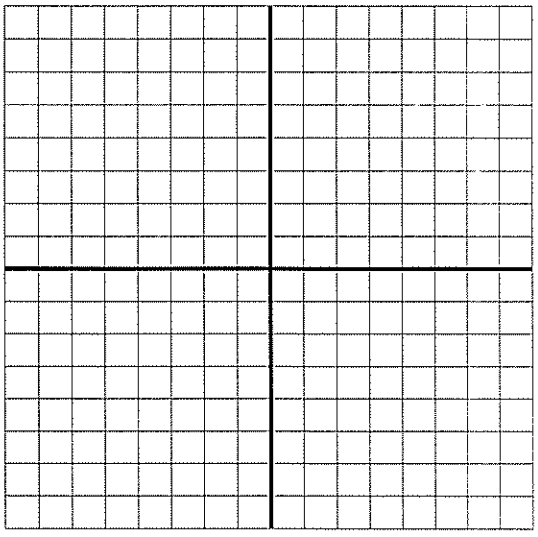


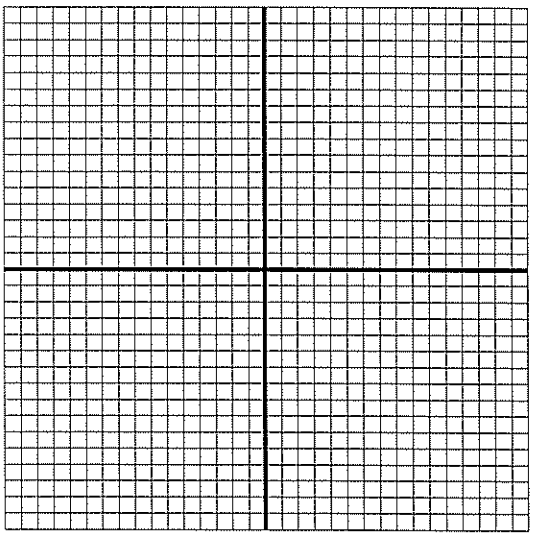




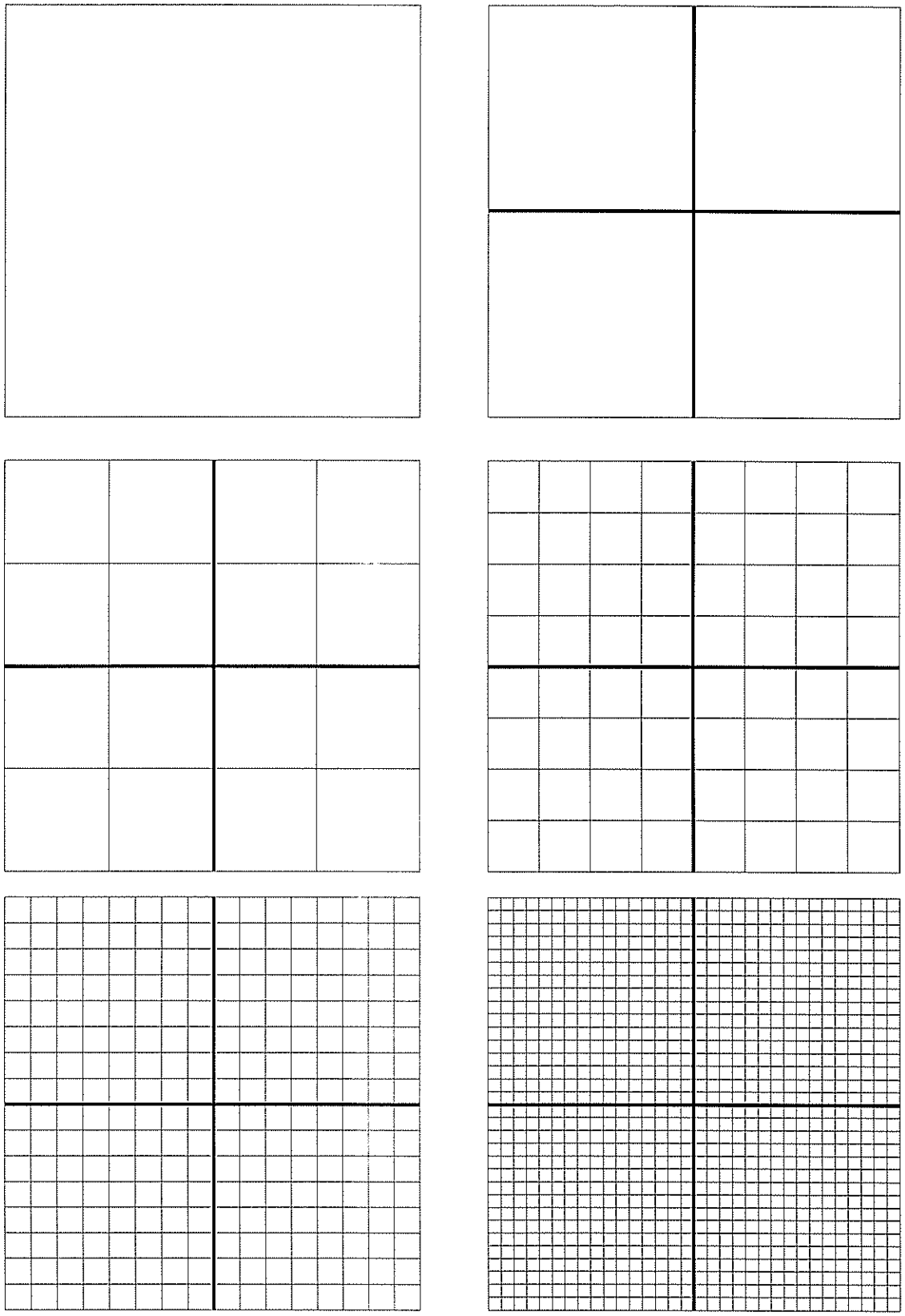






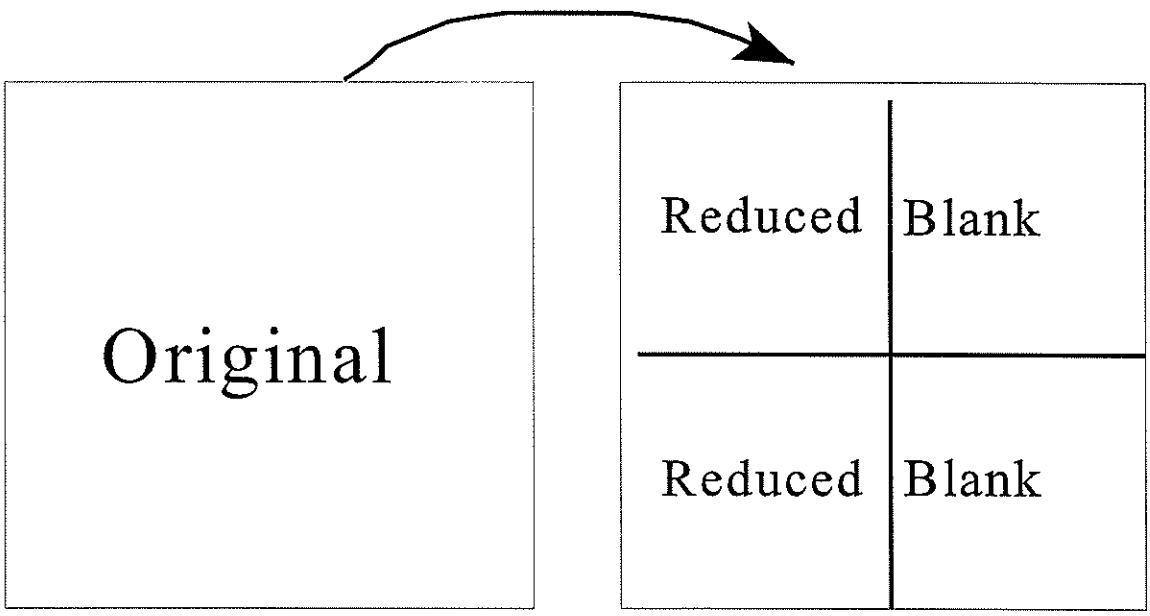


GRIDS FOR 2X2 REPLICATION



GRIDS FOR 2X2 REPLICATION

Iterating Instructions



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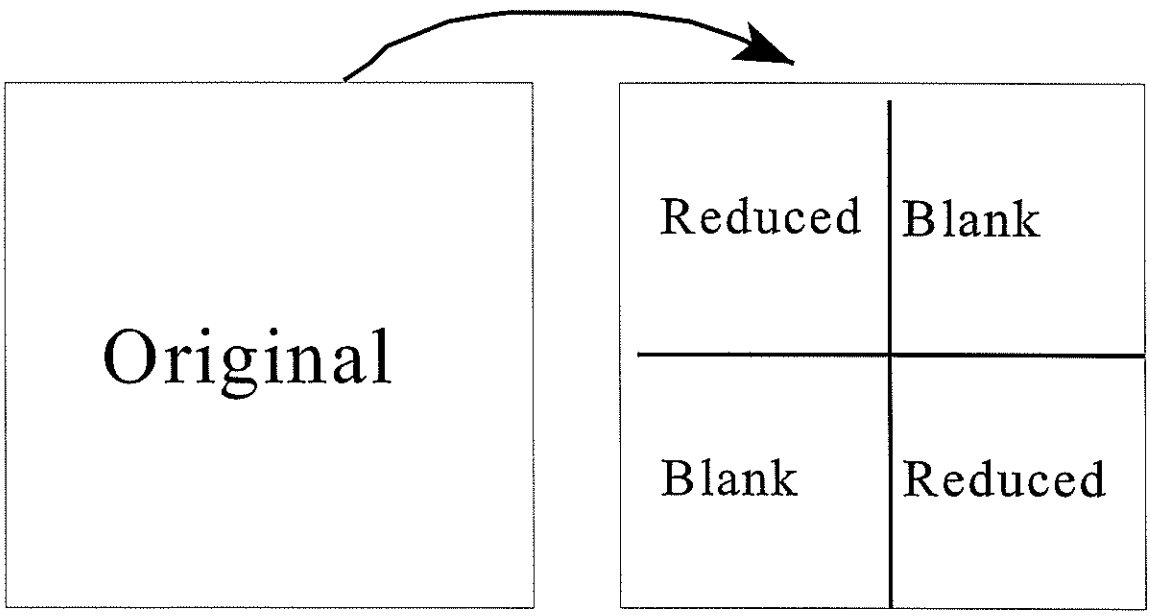

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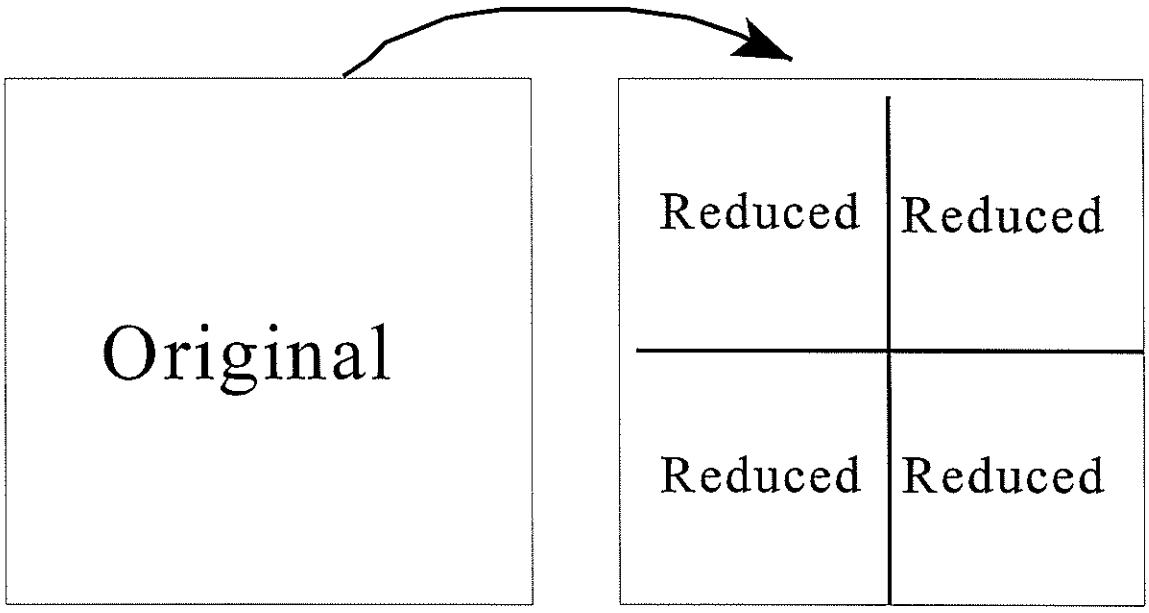

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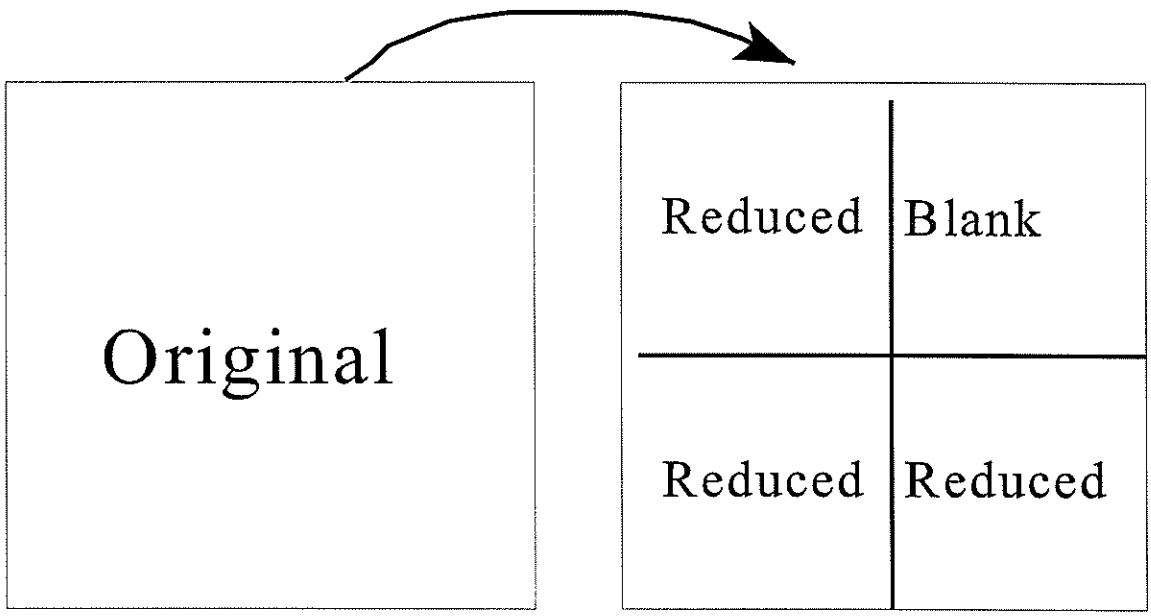
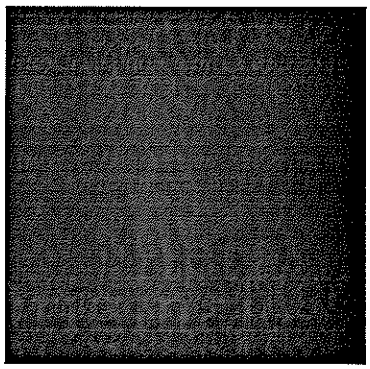
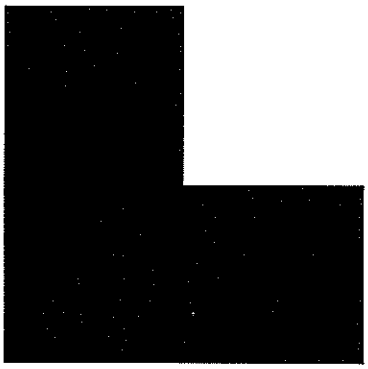
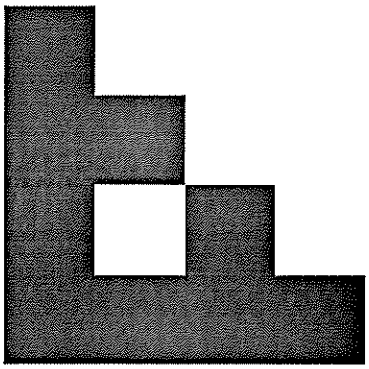




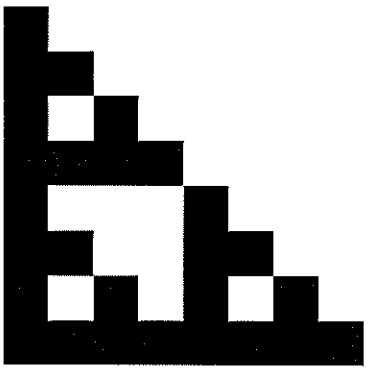
Stage 0



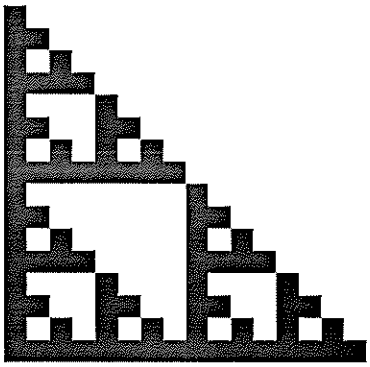
Stage 1



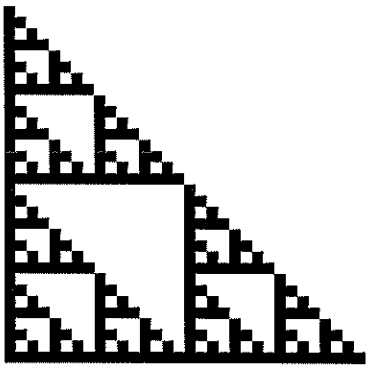
Stage 2



Stage 3

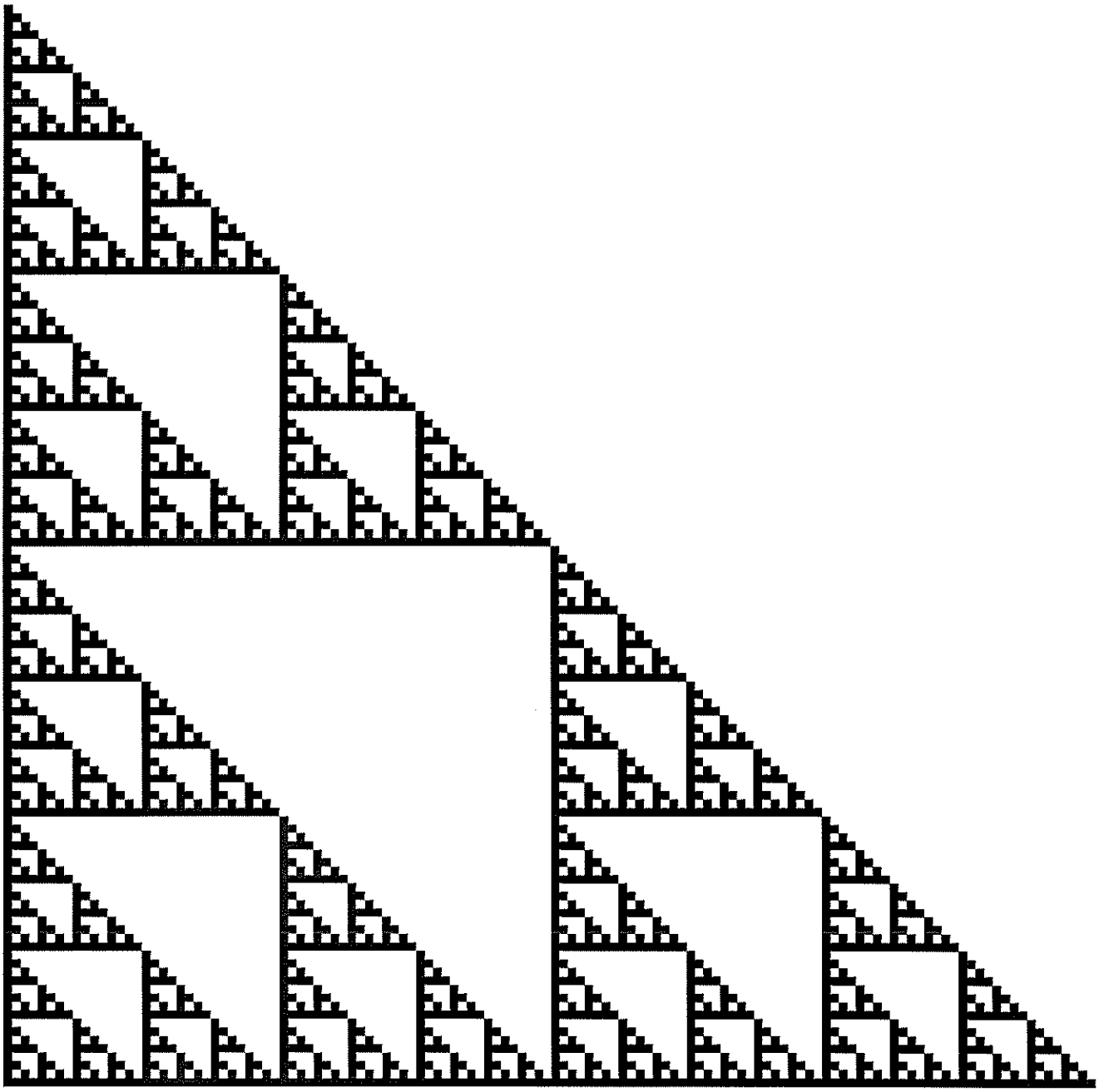


Stage 4

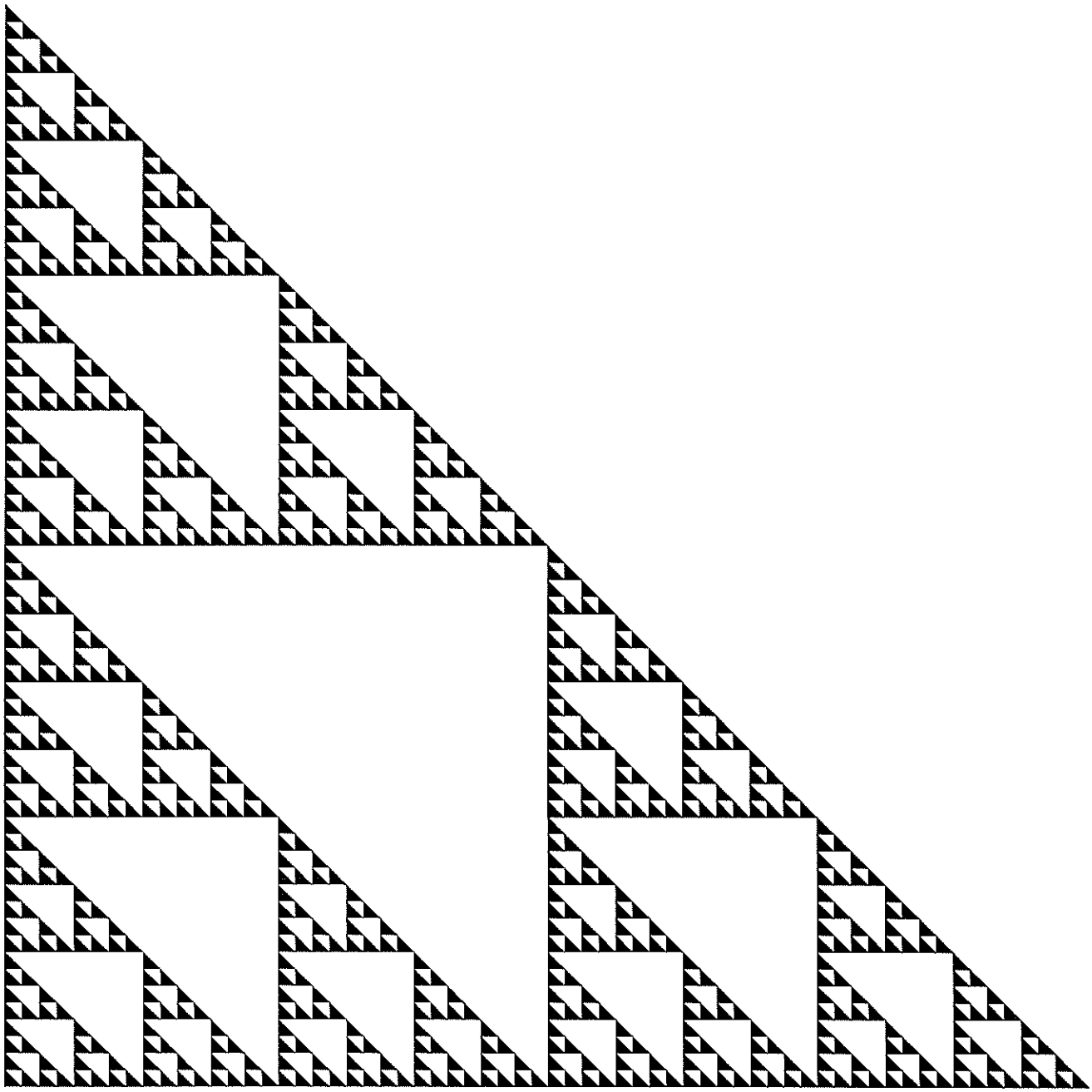


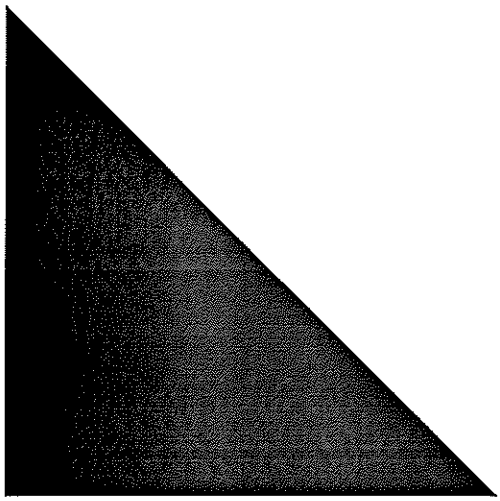
Stage 5

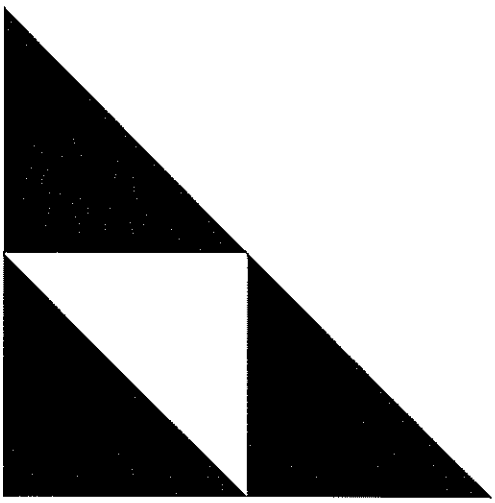
Stages 0-5 in the construction

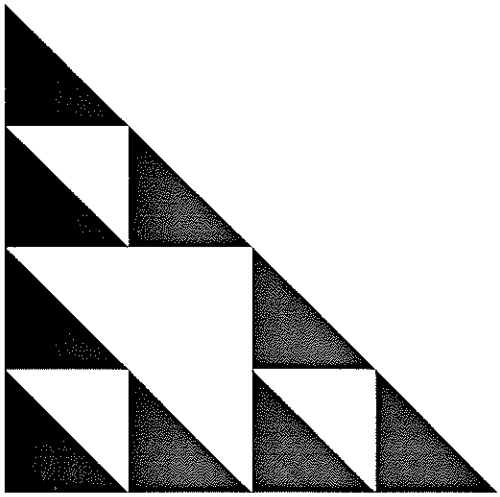


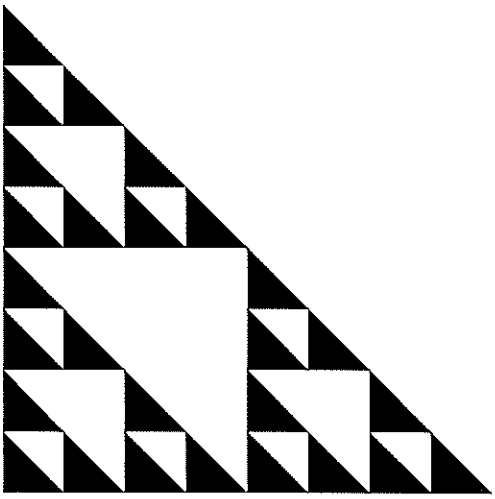
Stage 7 in the construction.

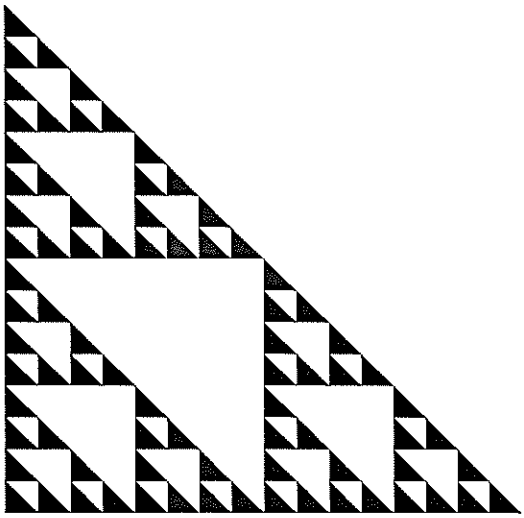


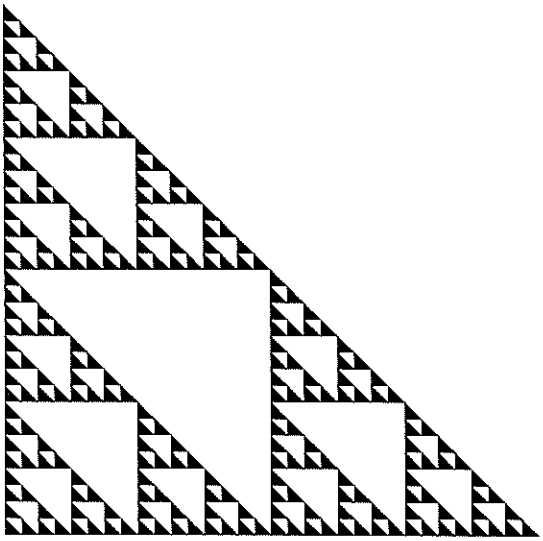


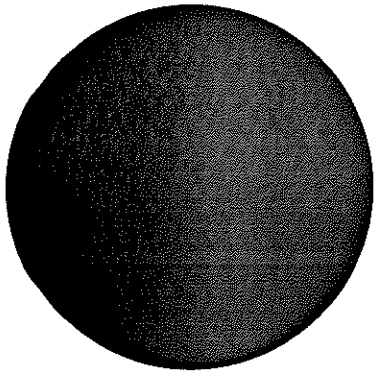


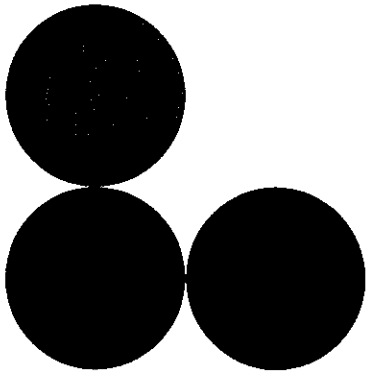


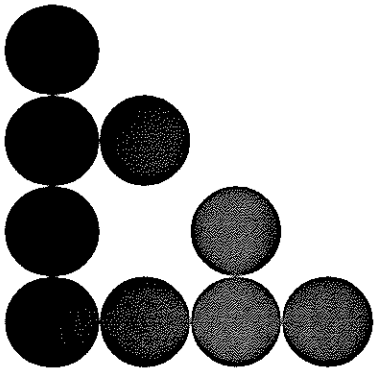


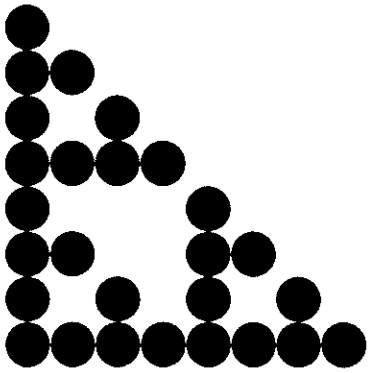


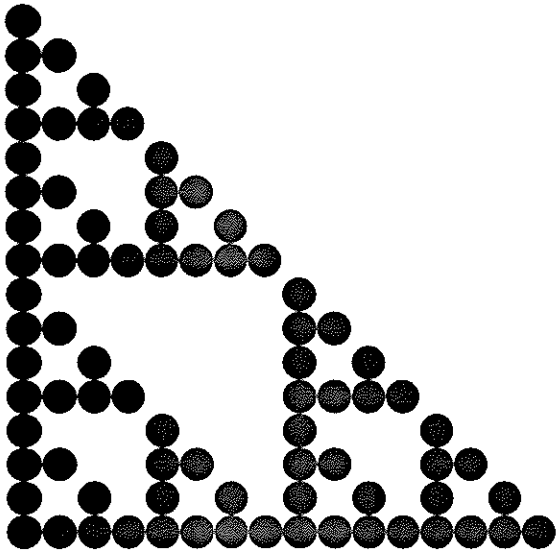


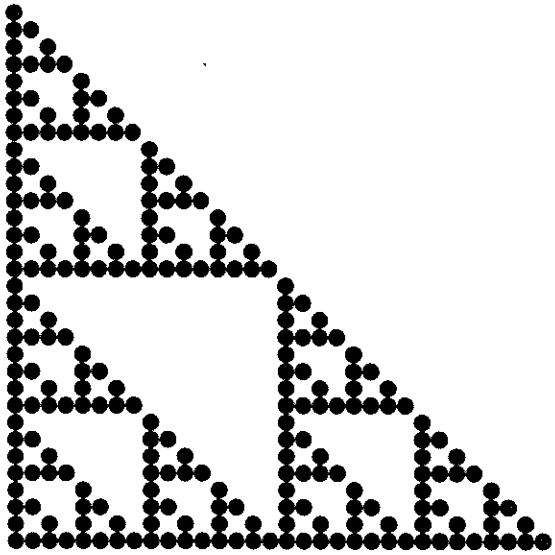


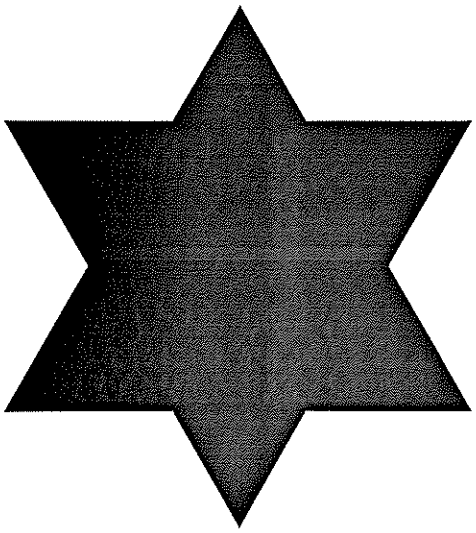


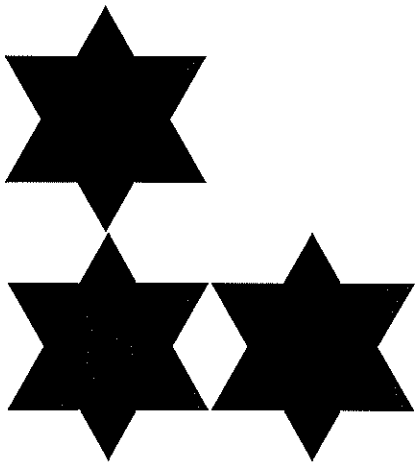


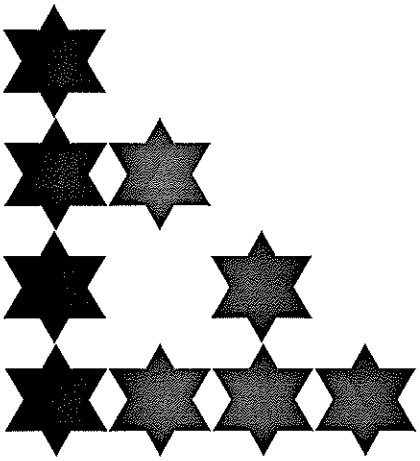


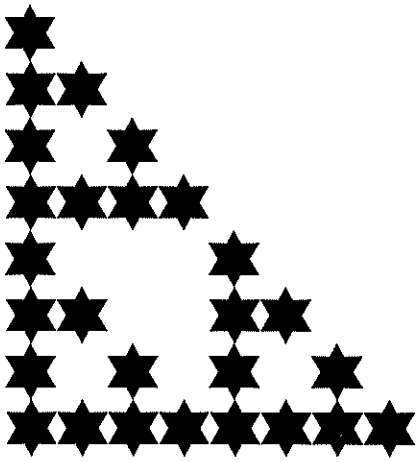


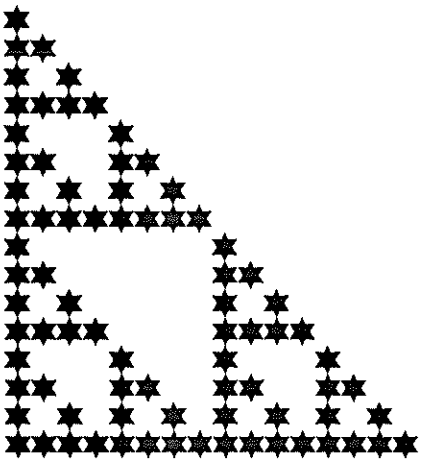


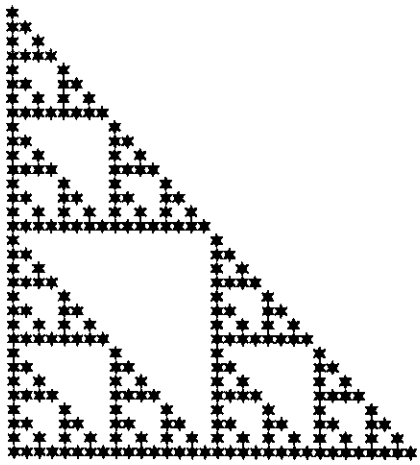


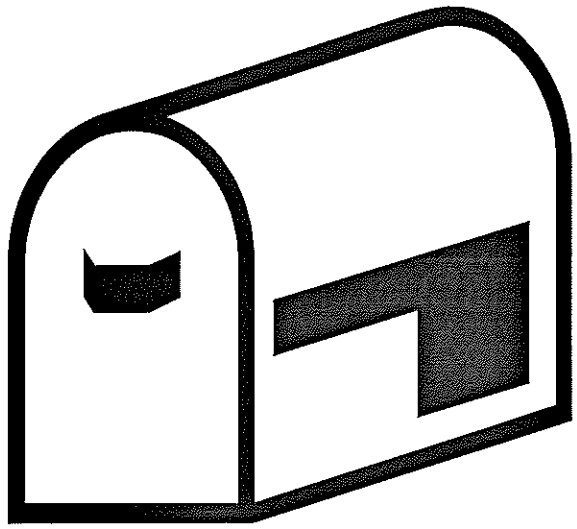


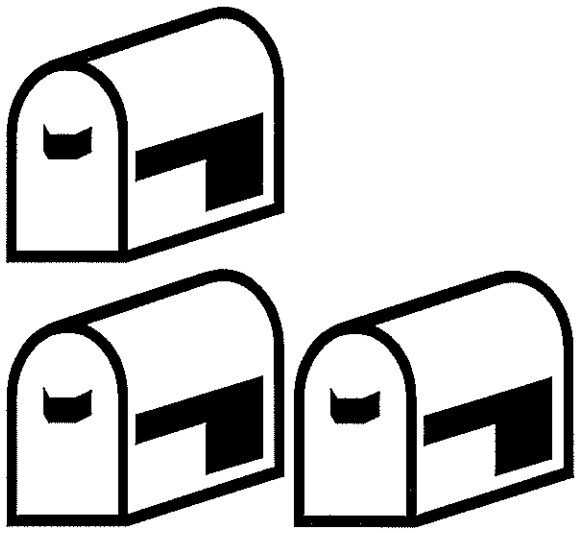


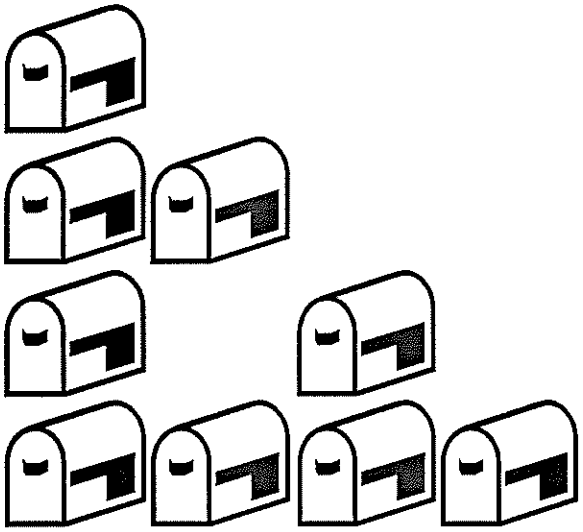


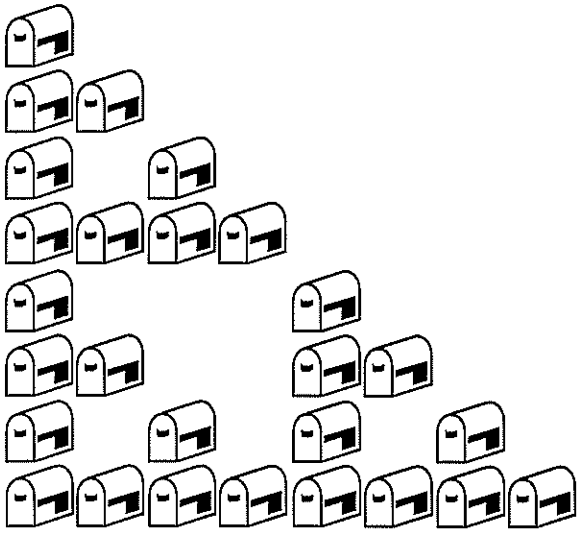


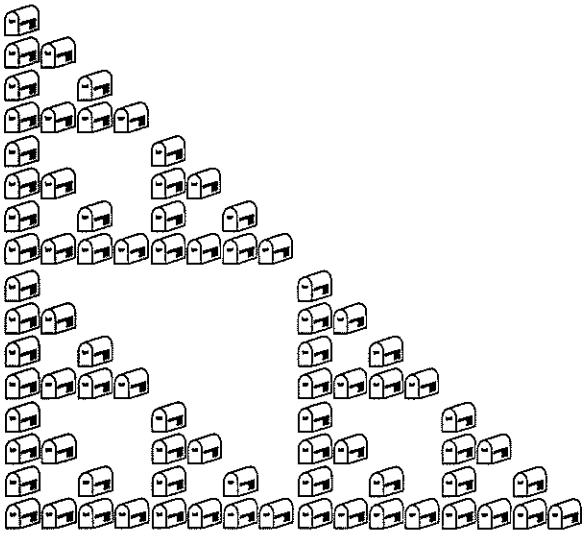




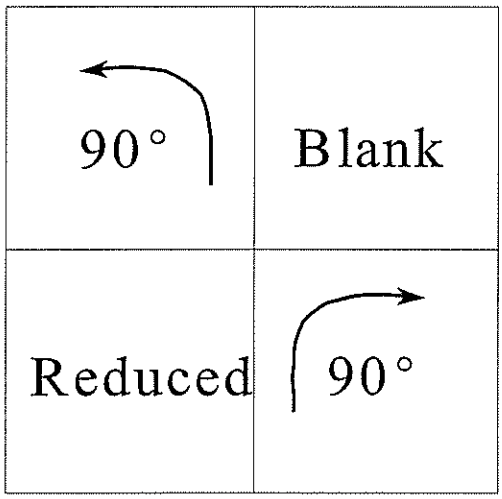








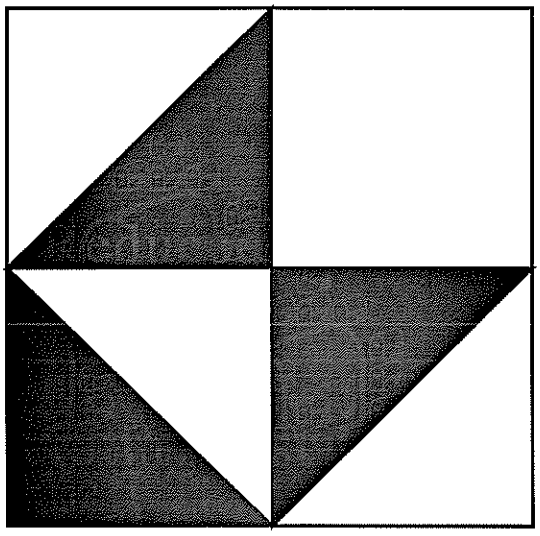
Yes, even Mailboxes!



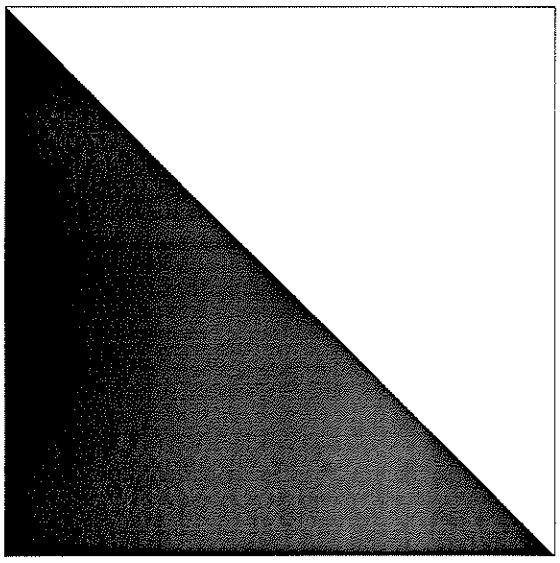
Iterating Instructions

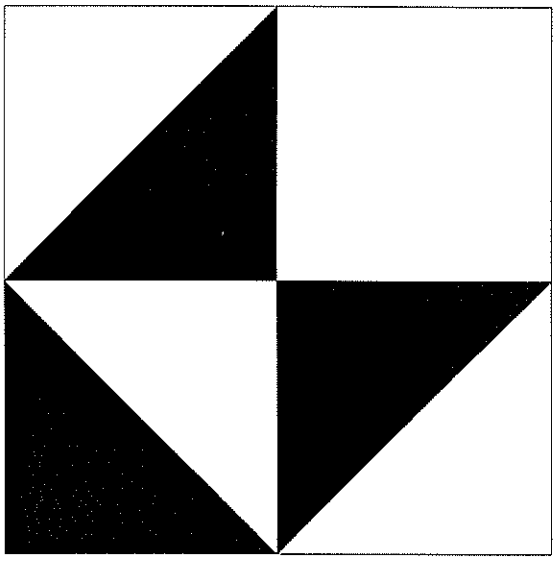
In this case, two of the reduced copies are rotated as they are placed in their positions.

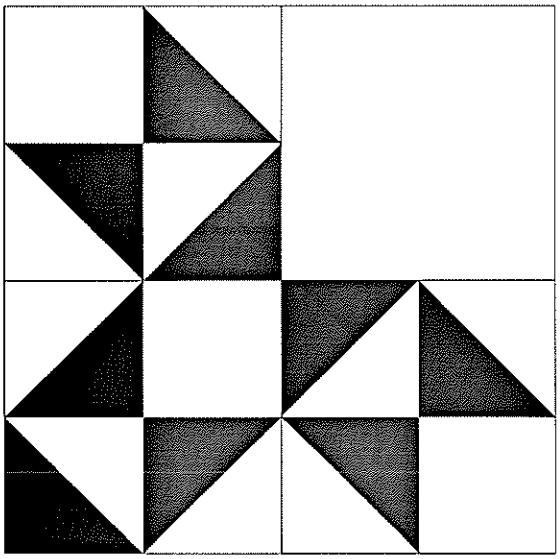
The figures to the left show the rule.

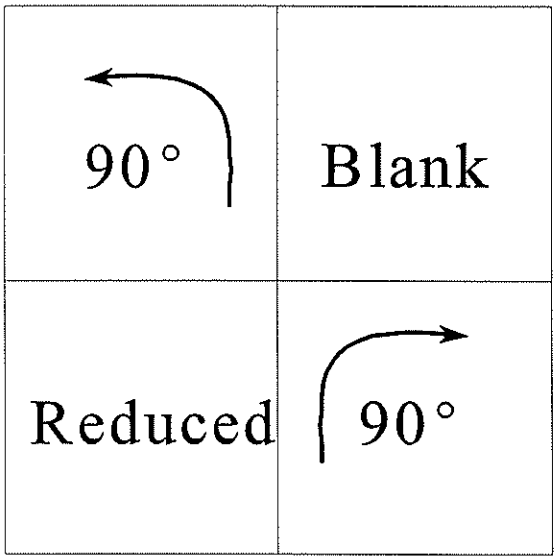


In the pictorial version of the instructions, we use triangles instead of squares so that the rotations can be seen.









Iterating Instructions

REDUCE, REPLICATE, REBUILD
Rotations in Square Grids

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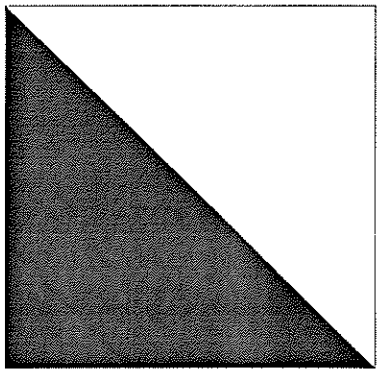
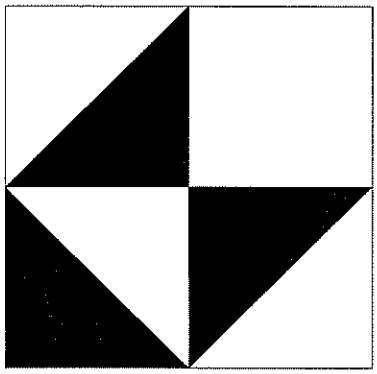
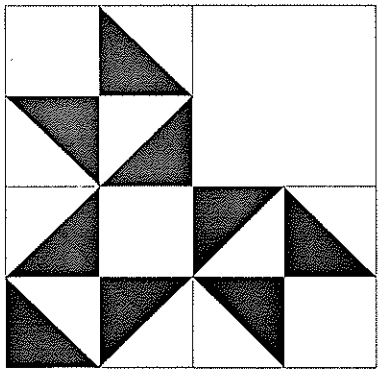
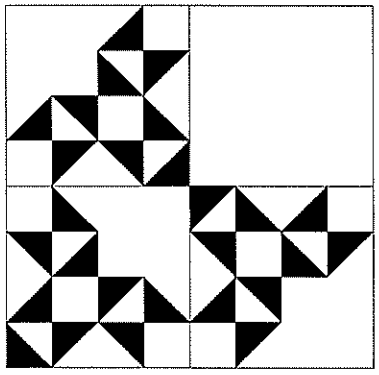
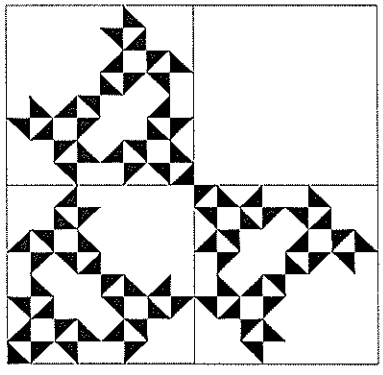
Montclair State University

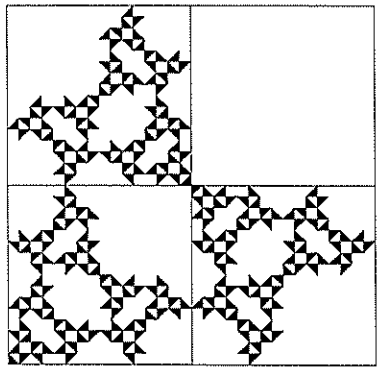


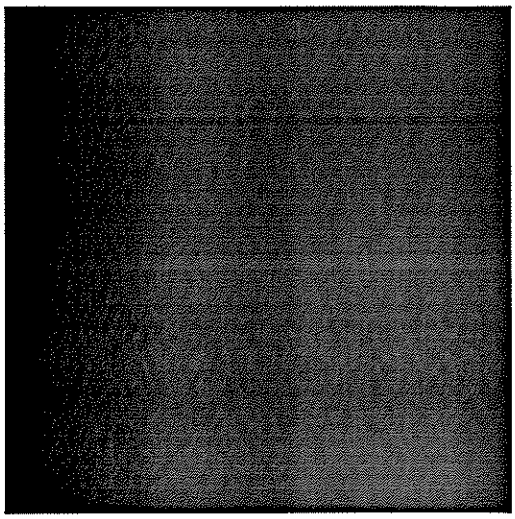


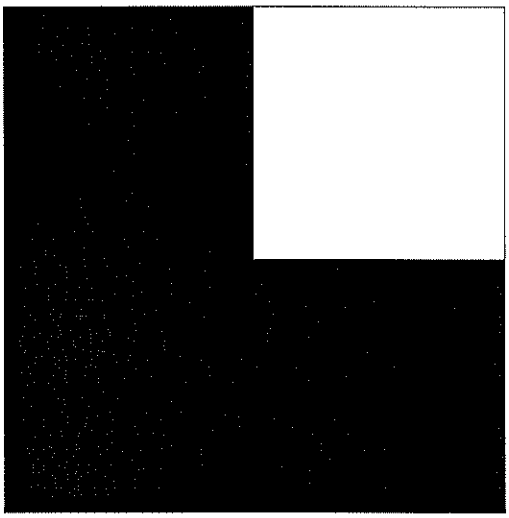


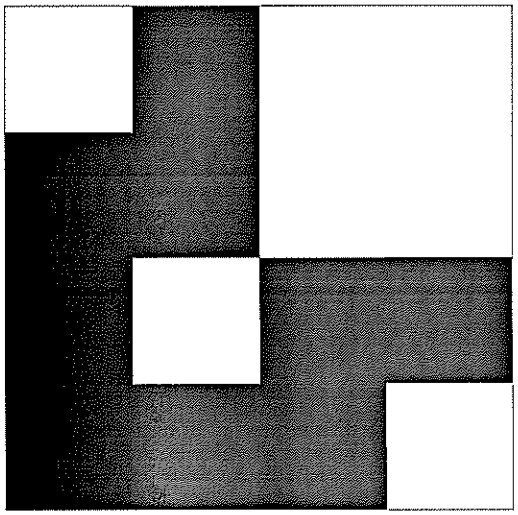


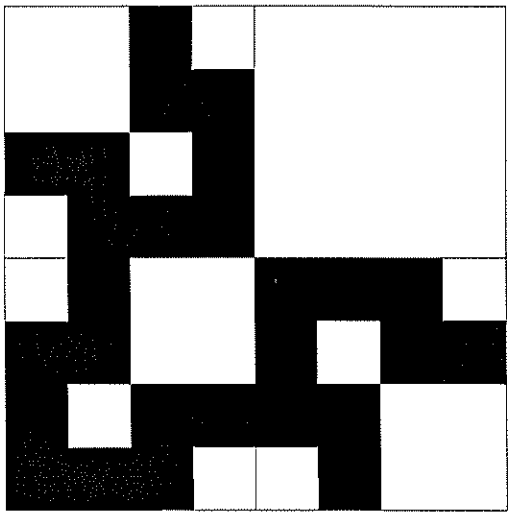


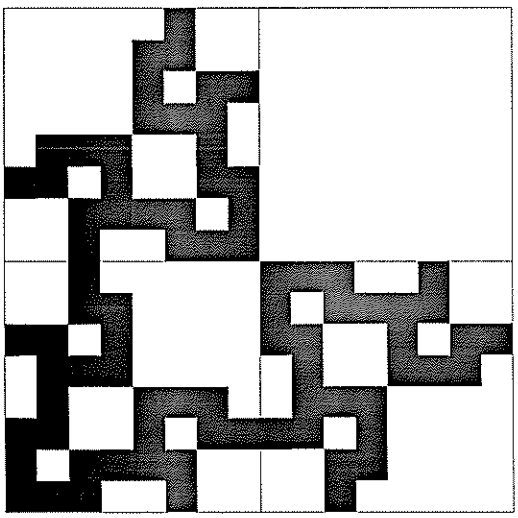


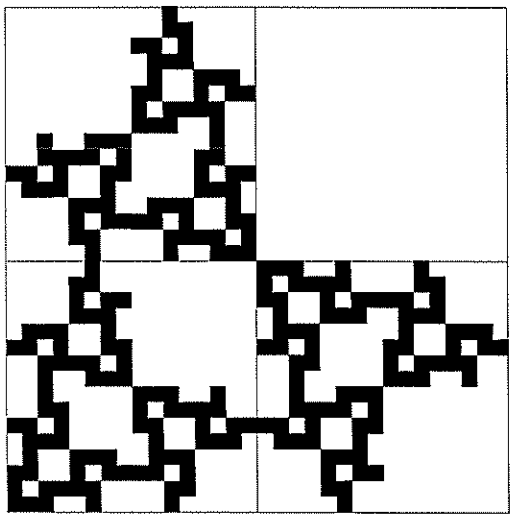












FRACTAL GENERATION
Repeated Copies

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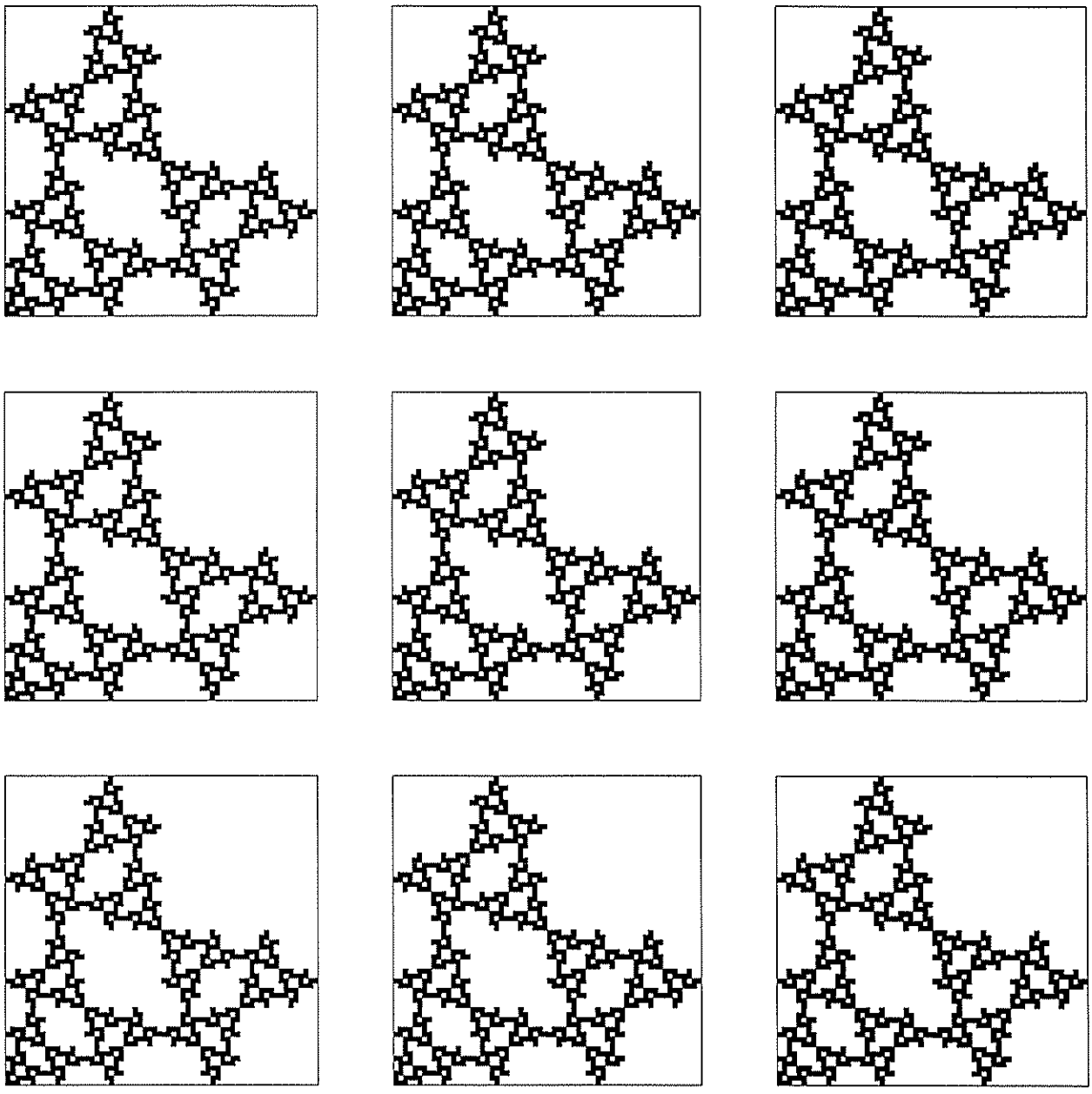
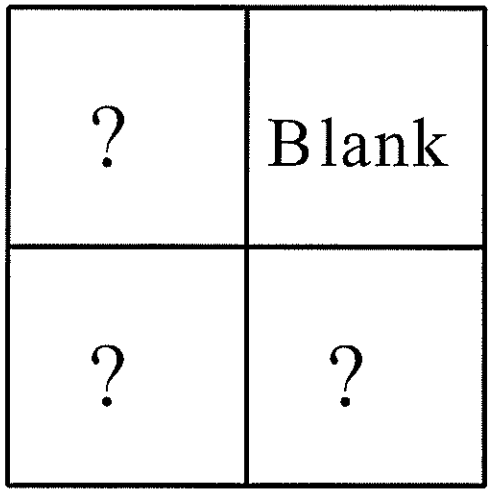
Montclair State University



Generating Various Fractals



How many different fractals can be generated if we:

- Reduce by 50%
- Replicate 3 times
- Rebuild by placing a copy in each of the indicated quadrants, leaving the upper right quadrant blank

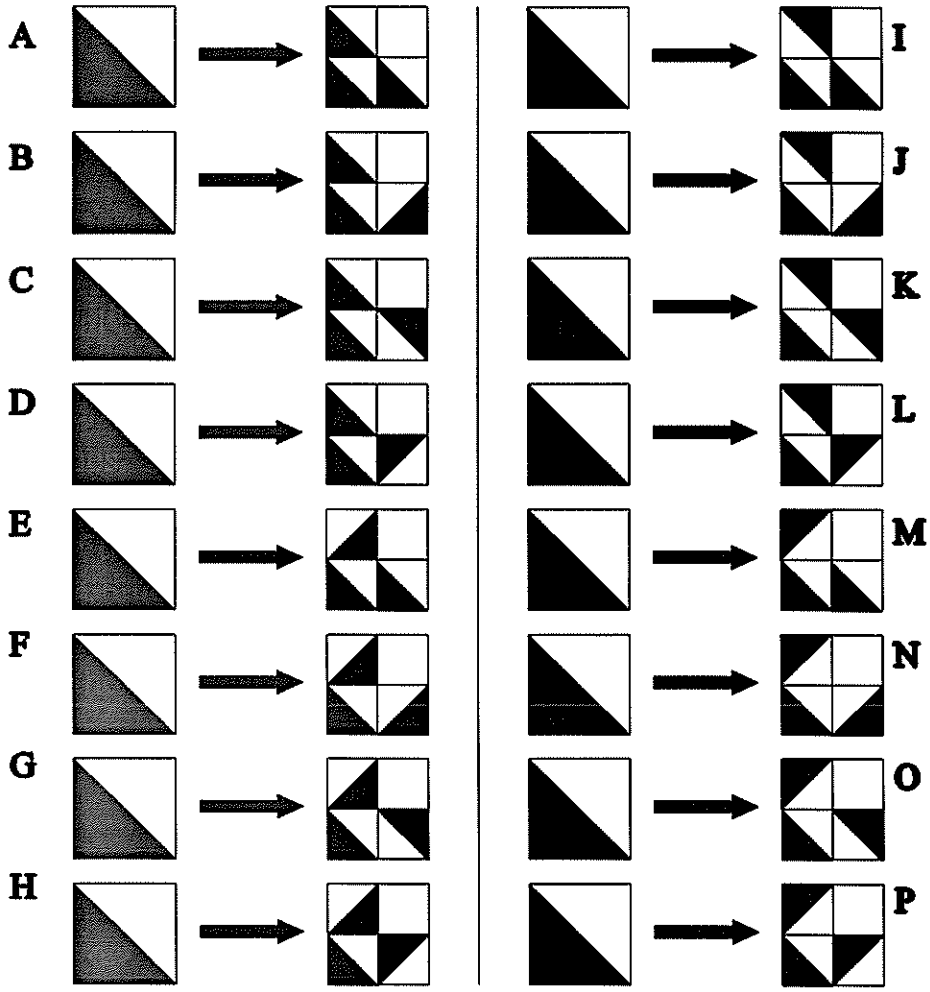
Variations for the rebuild step

If we generate a fractal by starting with a right triangle (so that we can see the rotations), and at each stage:

- Reduce by 50%
- Replicate 3 times, and
- Rebuild by placing an unrotated copy in the lower left quadrant, and a rotated copy in each of the lower right and upper left quadrants

then there are 16 possibile rebuilding rules. These are all shown below.

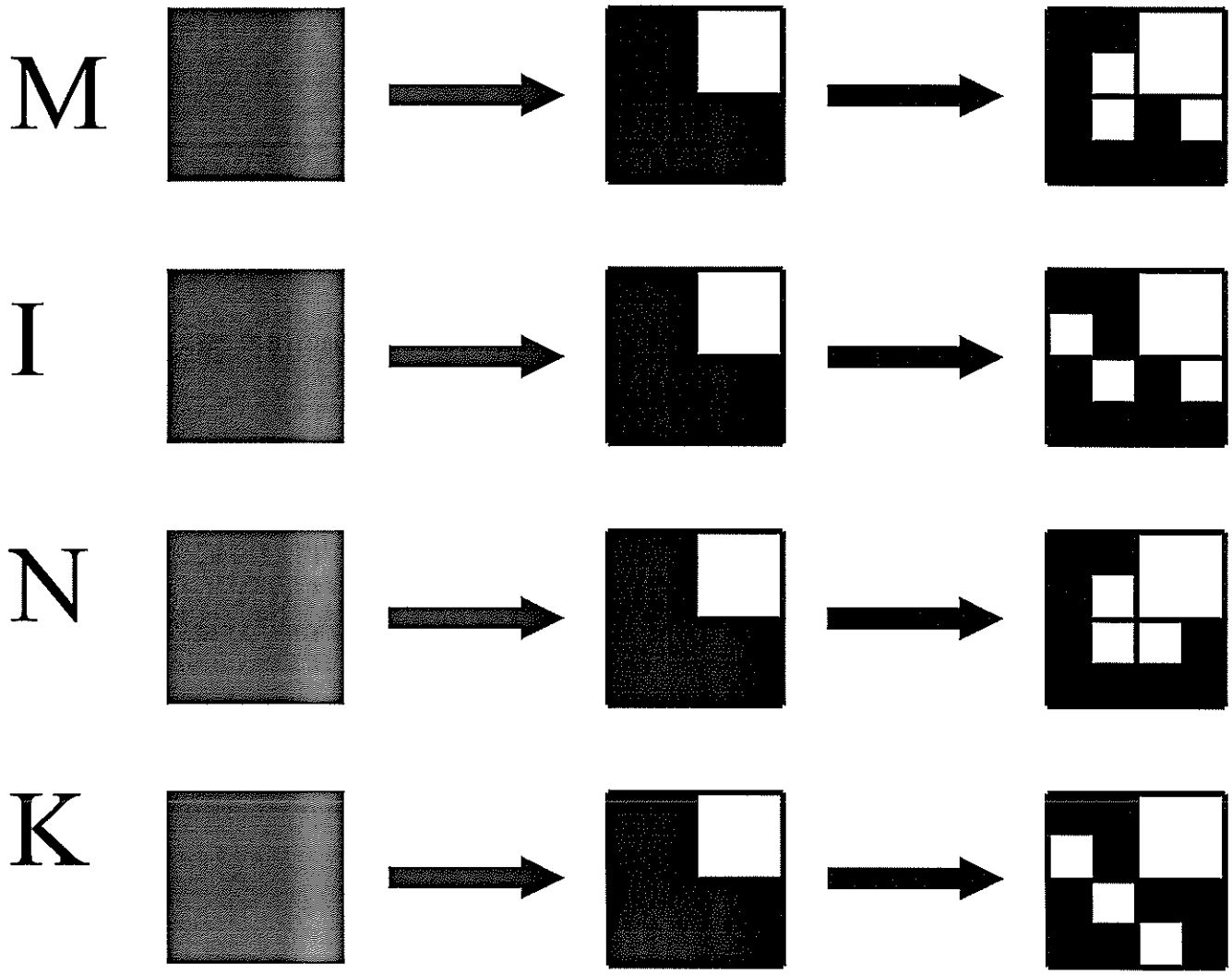
Note that "A" yields the Sierpinski triangle, and "H" yields the one we built during workshop.



Rot	**Blank**
Orig	**Rot**

Handout #3 — The MINK Rules

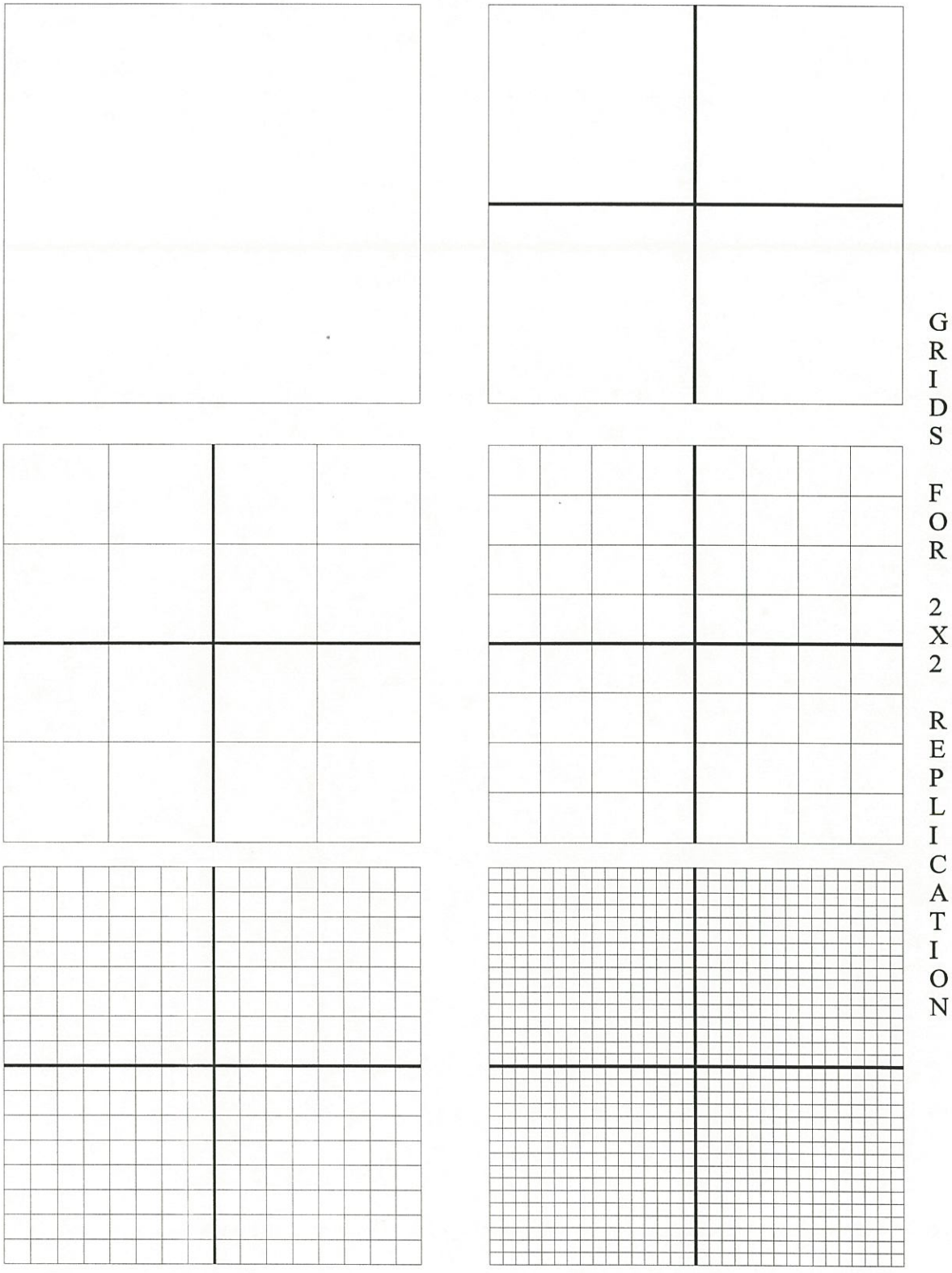
Here are four of the rules shown on the previous slide.



Build stages 0 - 5 of these figures. Note that we use squares instead of triangles since they are much easier to draw in the grids.

Handout #~~4~~ — ~~Reduce, Replicate, Rebuild~~ Generate Your Very Own Fractal

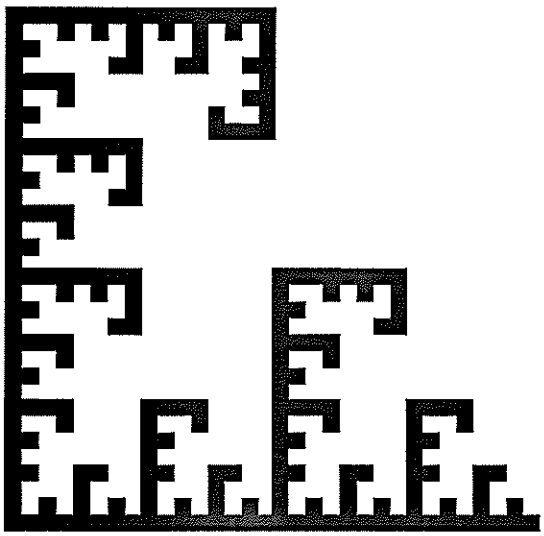
One of the rules from the previous handout will be assigned to your group. Use that rule to generate the first 4 or 5 stages of your Very Own Fractal.

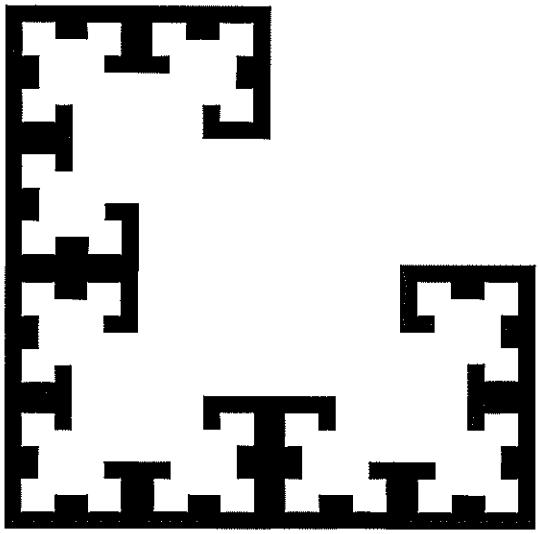


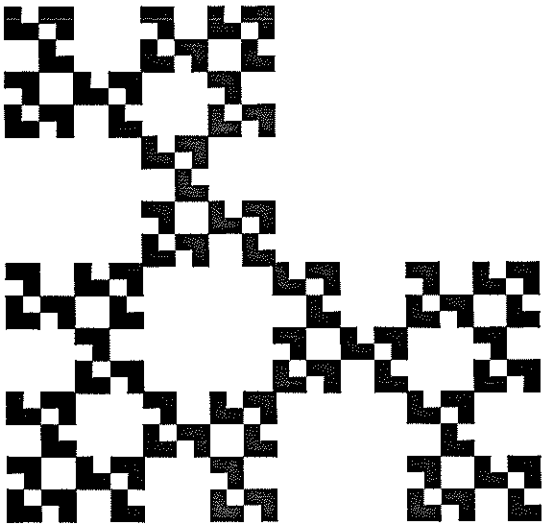
GRIDS FOR 2X2 REPLICATION

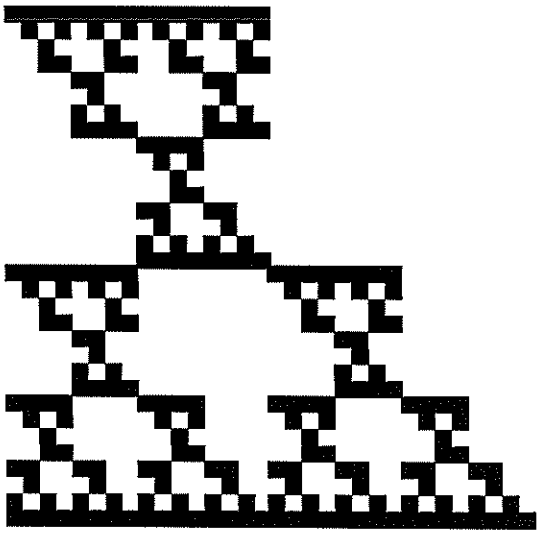
Matching MINKs

Can you figure out which of the rules, M, I, N or K, generated each of these figures?

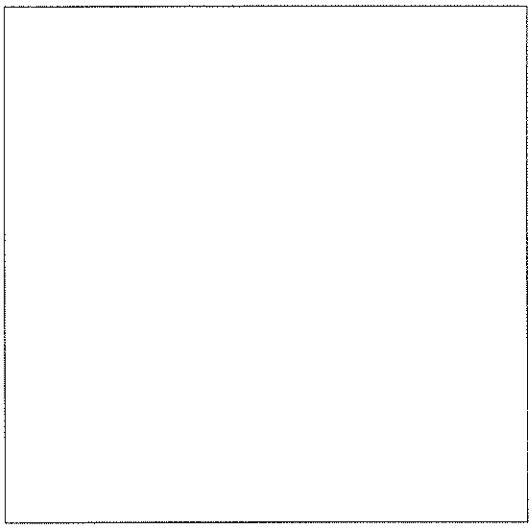


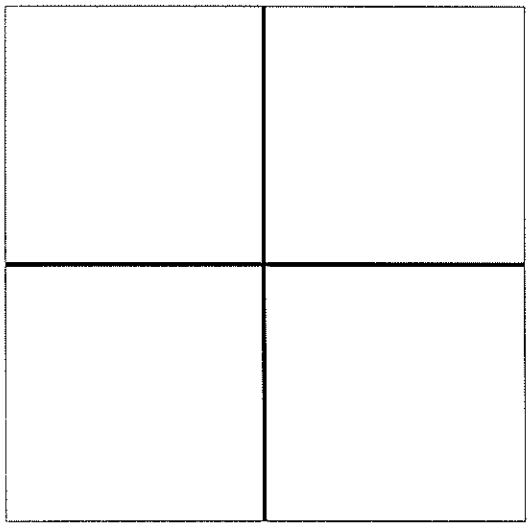


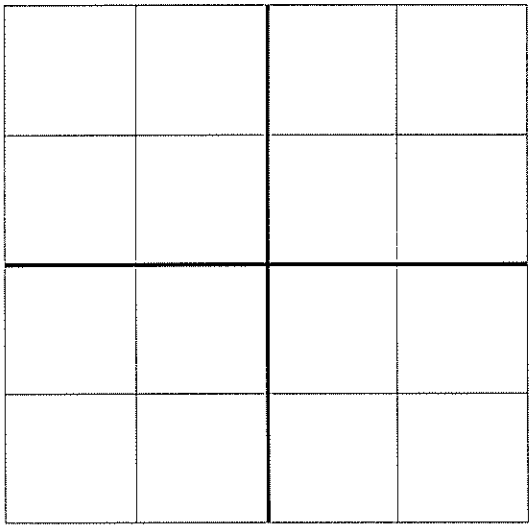


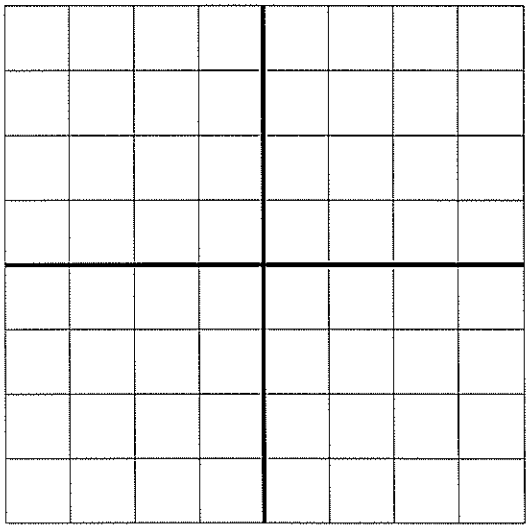


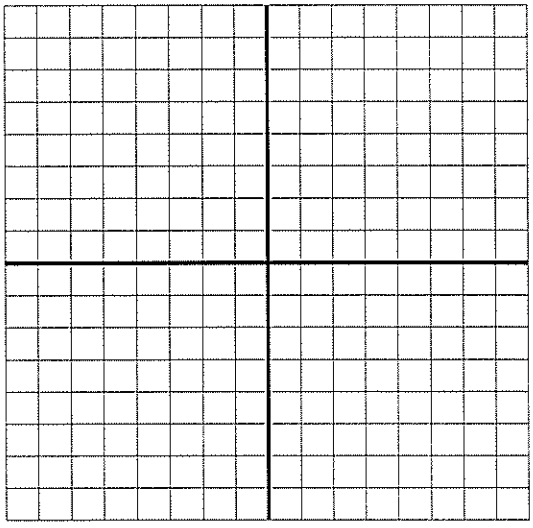
Handout #1 — Reduce, Replicate, Rebuild

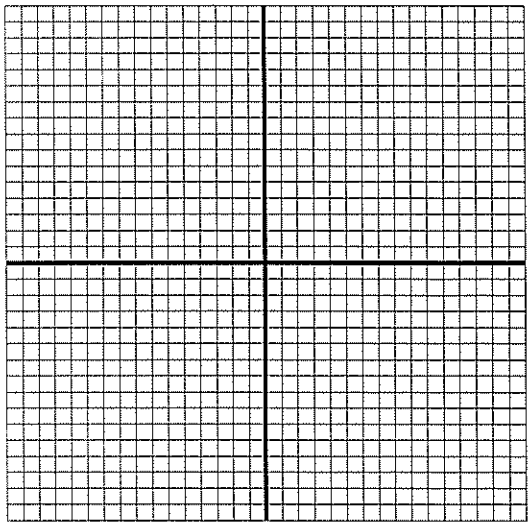












GRIDS FOR 2X2 REPLICATION

Handout #2 — Wallpaper Fractal

FRACTAL GENERATION
Repeated Copies

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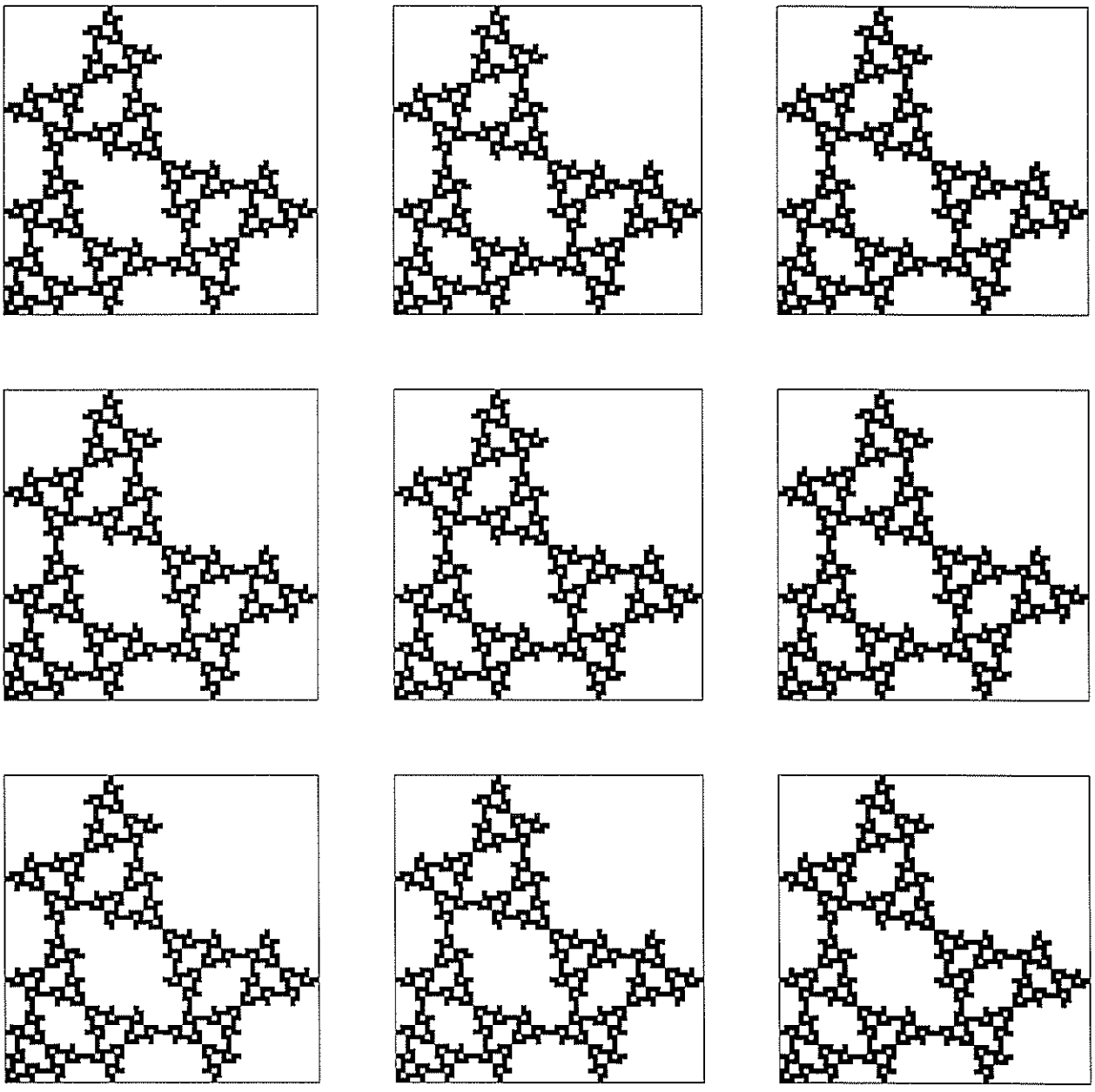
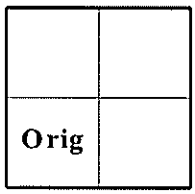
Montclair State University

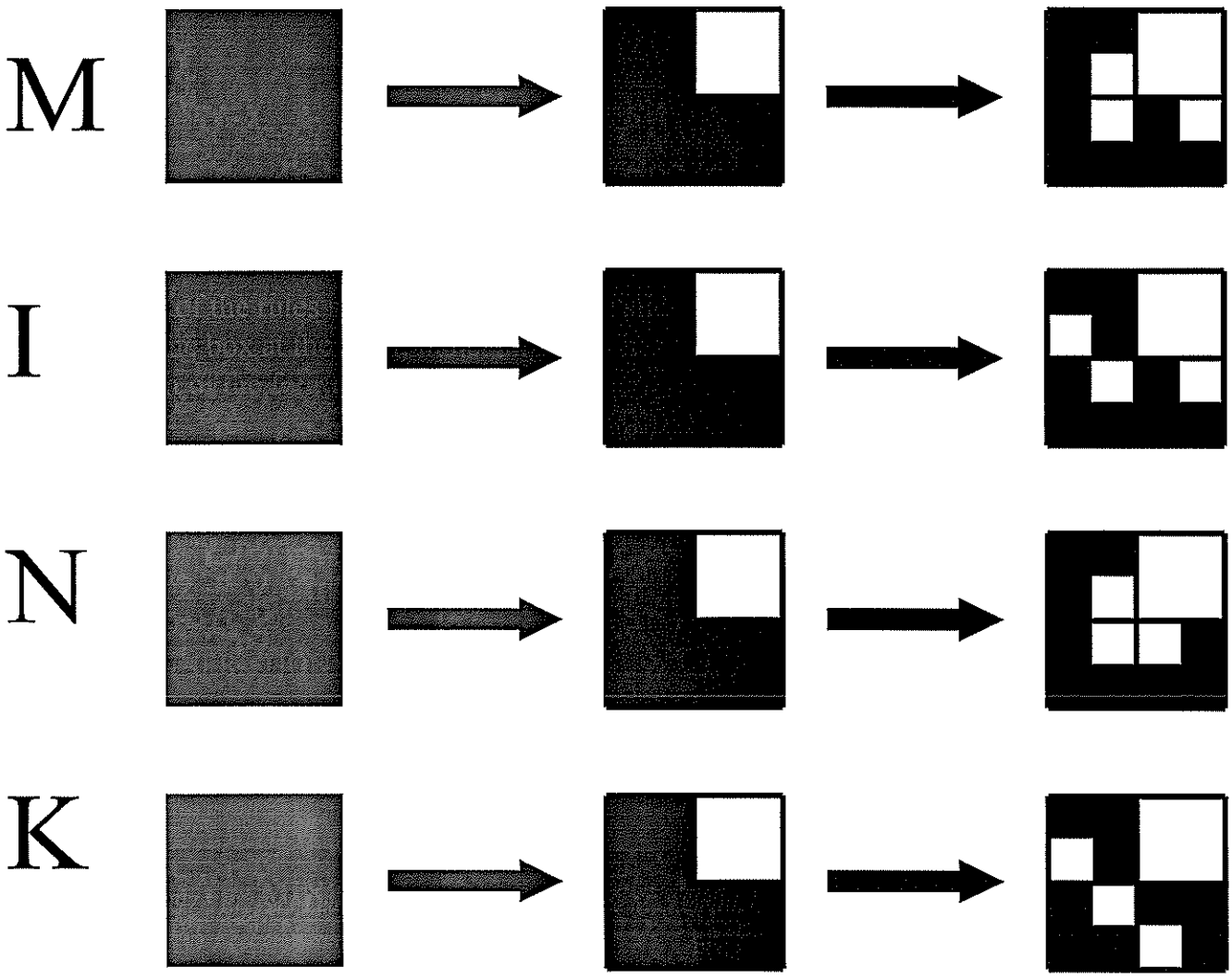


Handout #3 — The MINK Rules

Here are four of the rules shown on the previous slide. Write your rule explicitly in the box at the right, indicating what has to be done to the solid square in each of the four quadrants.

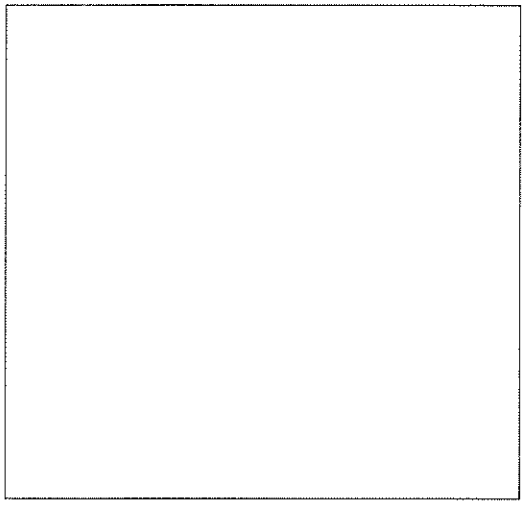


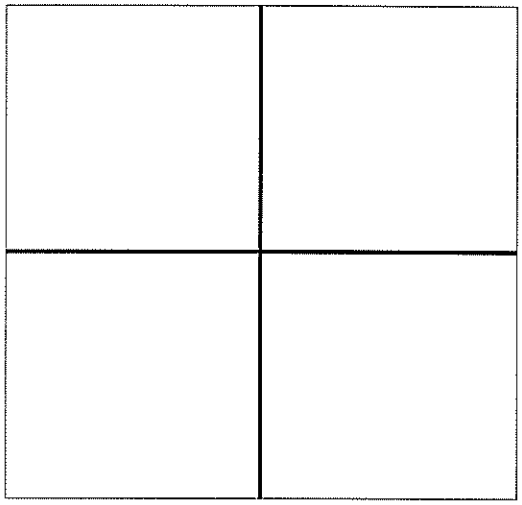
Your group will build one of these through the fifth stage. Note that we use squares instead of triangles since they are much easier to draw in the grids.

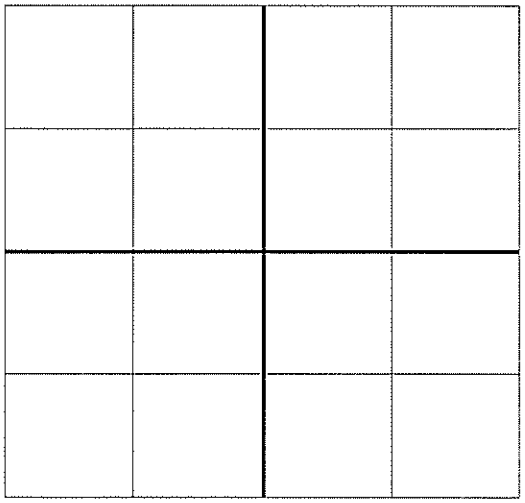


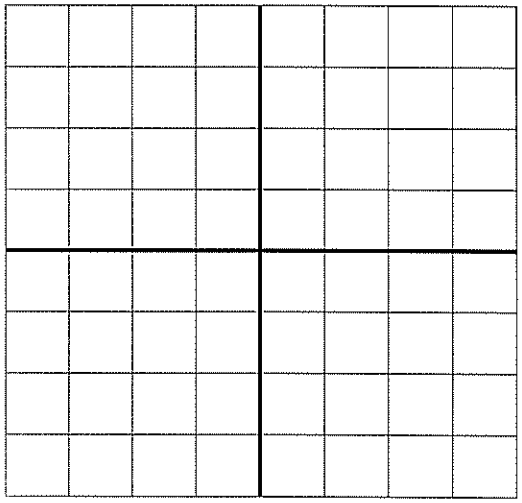
Handout #4 — Generating our Very Own Fractal

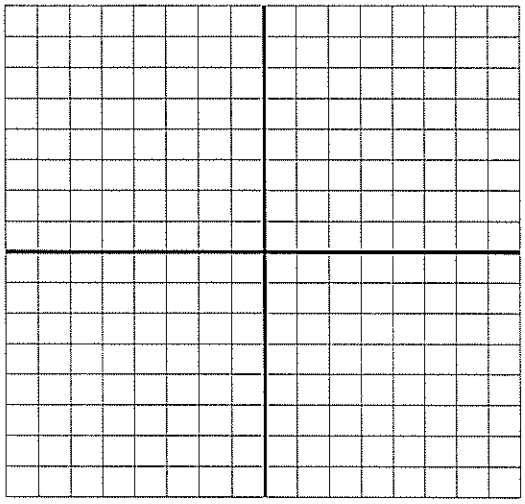
One of the rules from the previous handout will be assigned to your group. Use that rule to generate the first 4 or 5 stages of your Very Own Fractal.

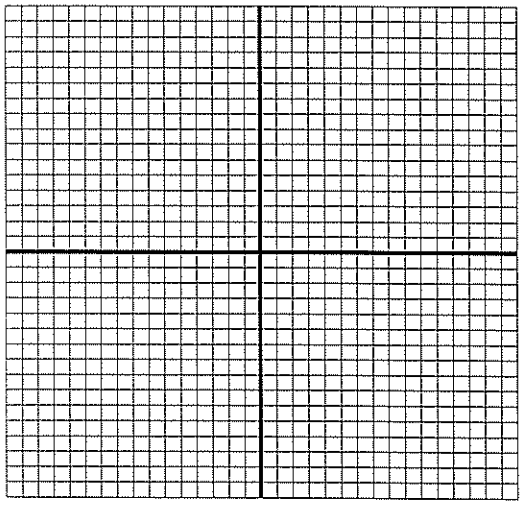












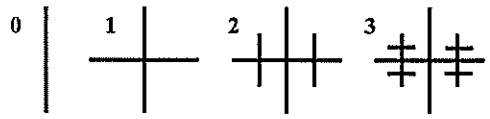
GRIDS FOR 2X2 REPLICATION

Workshop 10 — Generating Fractals — Exercises

Study Group Problems:

1. Consider the two sets of figures on the next page. Describe the building process in terms of reduction, replication and rebuilding.

2. Describe, in words, the iterative rule for adding line segments used to generate this sequence of figures. Note that this rule can be described in terms of reduce, replicate and rebuild, and that a clue can be gotten by considering the transition from stage 2 to stage 3. Then use the dot paper provided to draw the next one or two figures. (Stages 0, 1, 2 and 3 are shown.) What is a good length to choose for the first line you draw?

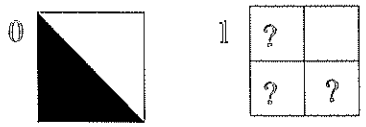


3. What is the minimum number of line segments needed to draw each of the figures in problem 2? How many line segments at stage *n*? Assume that the line segment in stage 0 above has length 1. What is the total length of all the segments in each of the other 3 stages? In the *n*th stage?

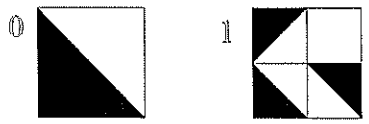
4. Stages 0 and 1 of an iterative building procedure are shown here.

   a. Describe the algorithm in terms of reduction, replication and rebuilding, assuming that there were no rotations.
   b. An *unshaded region* is a region which is unshaded, and is complete enclosed by shaded regions. For example, stages 0 and 1 have no unshaded region, but stage 2 has one unshaded region, which is shaped like a single square. How many unshaded regions will there be in stage 3? Stages 4, 5 and 6? Stage *n*?

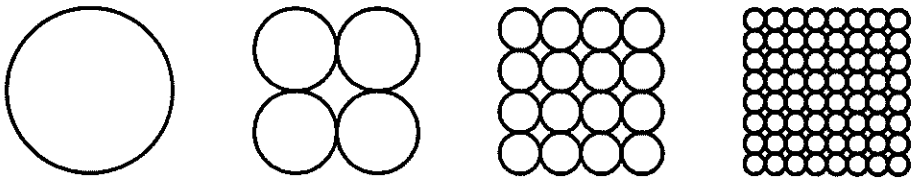
5. Construct your own building algorithm in this format using the three cells marked. Define the transformations in terms of rotations, specifying which rotation will be used for each cell, then draw stages 0 through 4 on the grids on Exercise Page 3 and 4. Describe the building algorithm in terms of reduction, replication and rebuilding. Then describe some characteristics you imagine the final fractal will have.



6. Study stage 1 of this building algorithm. Describe the reduction, replication and rebuilding. Be explicit in the rotations required in the rebuilding step with the reduced images. Draw stages 2 and 3 on the grids supplied.



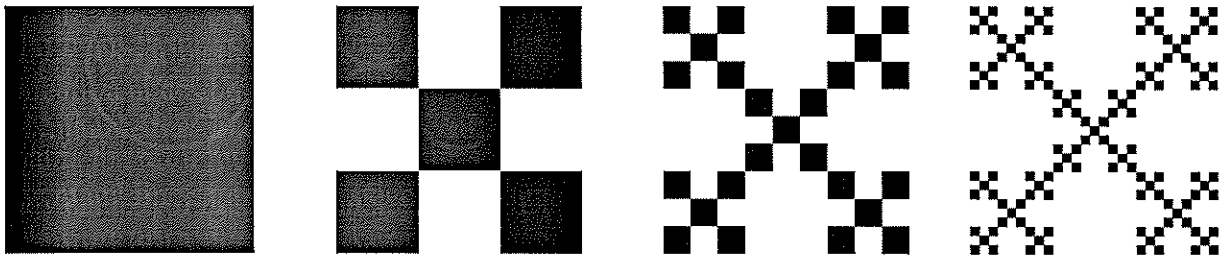
REDUCE, REPLICATE AND REBUILD



REDUCE:_____

REPLICATE:_____ TIMES

REBUILD:_____

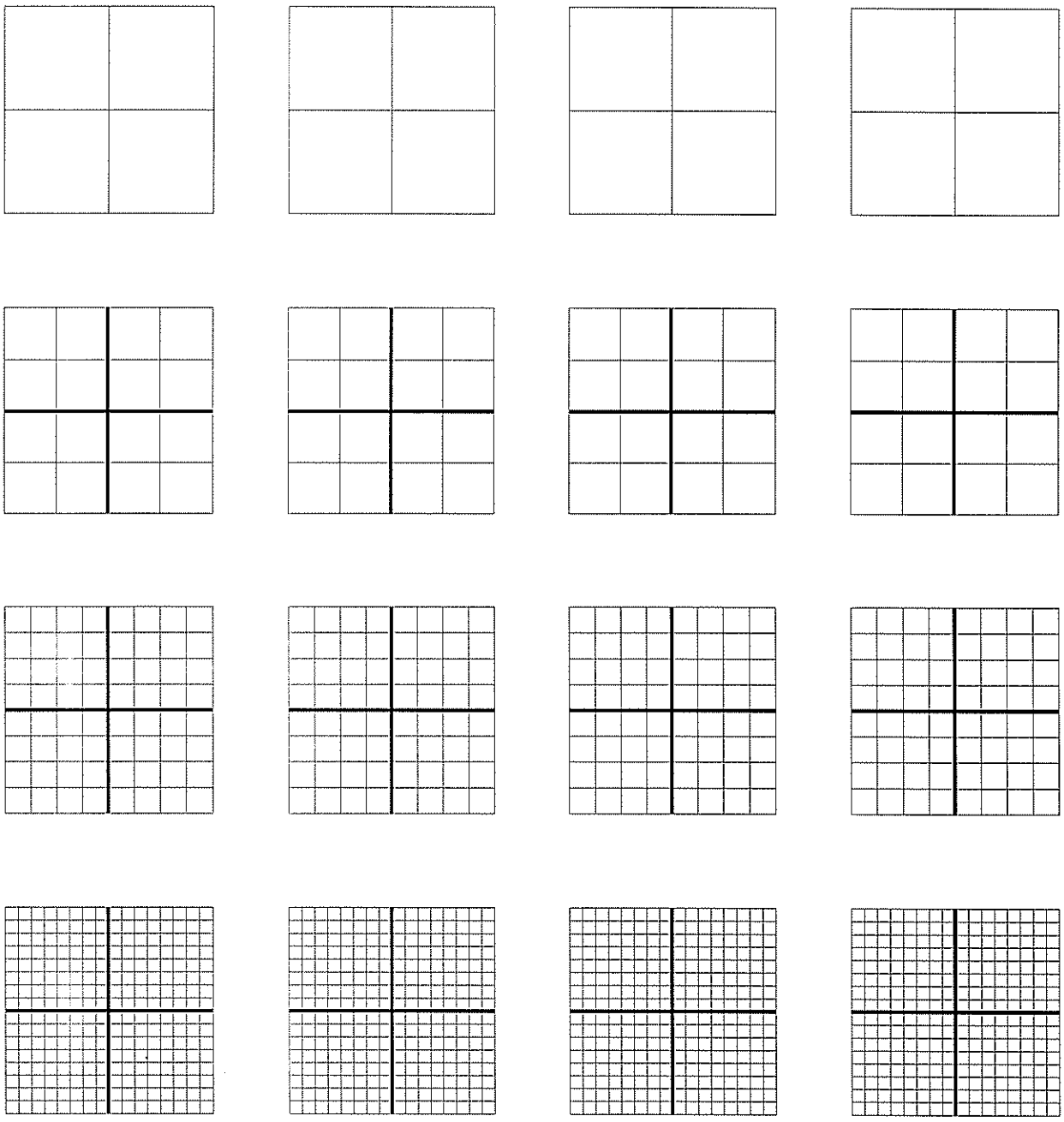


REDUCE:_____

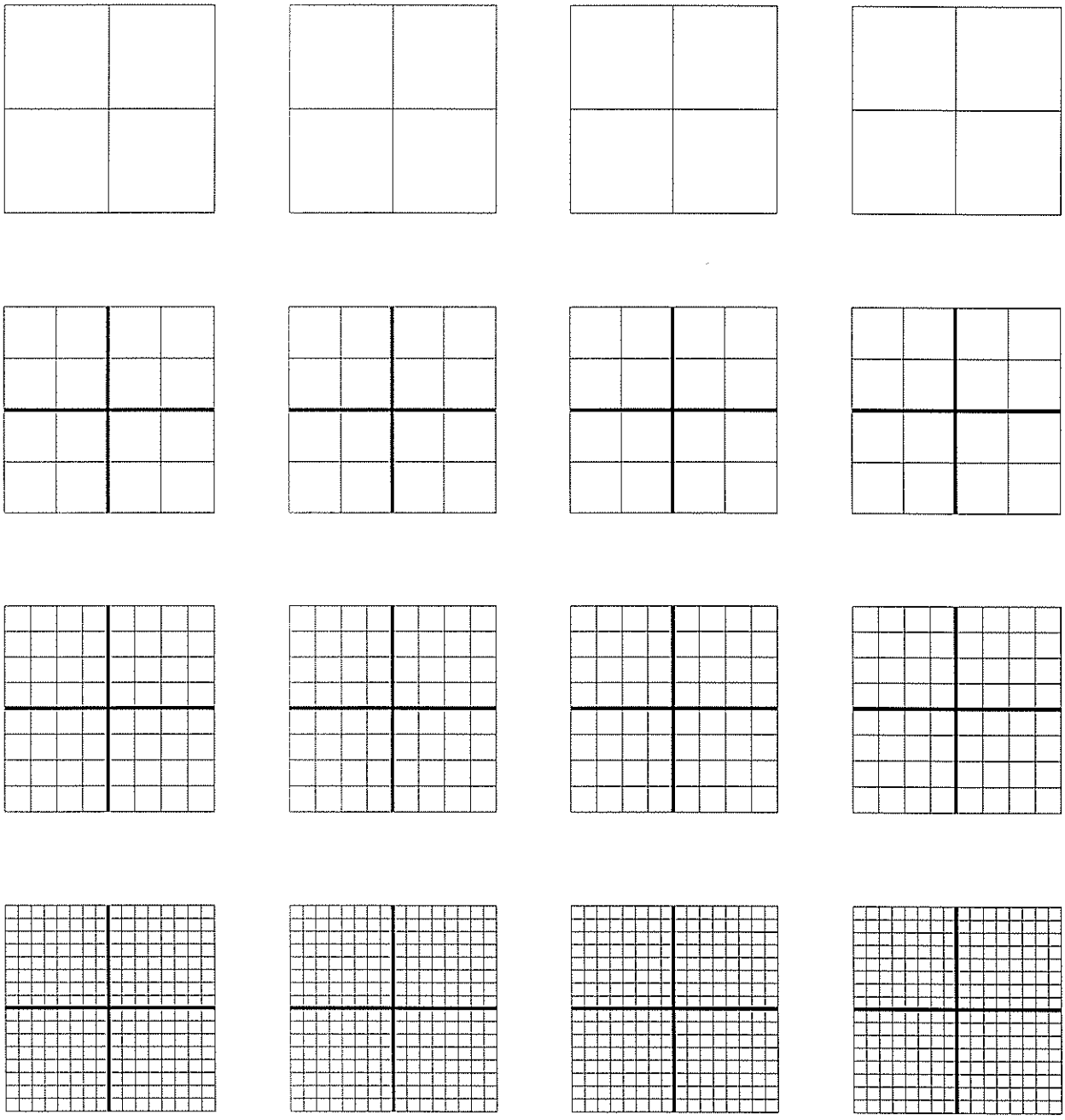
REPLICATE:_____ TIMES

REBUILD:_____

Grids for making fractals in 2x2 squares.



Grids for making fractals in 2x2 squares.



Resource Book

Workshop 10: Generating Fractals

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Workshop 10: Generating Fractals

Mathematical Background:

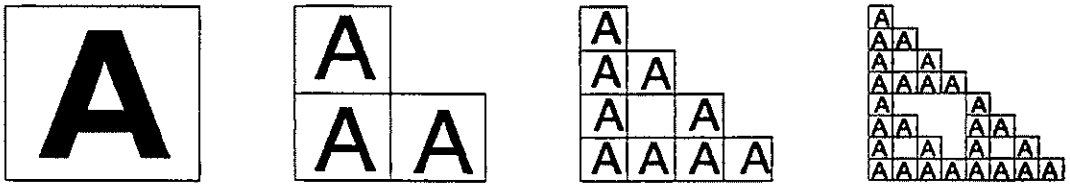
- The steps in a construction process that generate a fractal are recursive.
  Start with some initial figure as stage 0.
  Use it in a well defined procedure to build stage 1.
  Repeat the procedure with stage 1 to build stage 2.
  Repeat the procedure with stage 2 to build stage 3, and so on.

  In general, call up the figure at some stage n and use it to build the figure at the next stage, n+1.

- The iterative process that takes the figure from one stage to the next has three parts.

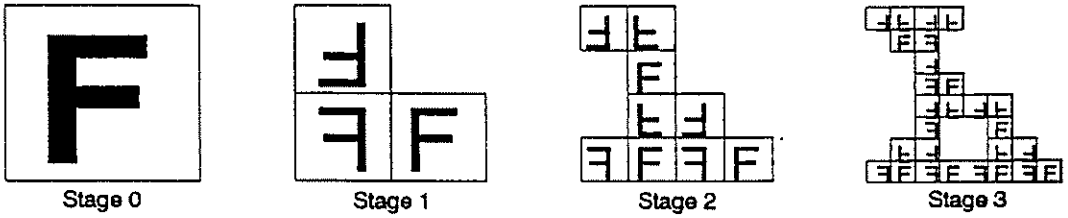
  Reduce Replicate Rebuild

  For the Sierpinski triangle, *reduce* to half linear size,
  replicate three times, and
  rebuild by placing the three reduced images back in the corners of the preceding figure without changing their orientation.



By changing the rebuilding step to include rotations or reflections, a new image emerges.

Notice here how each of the three reduced images of the entire figure is transformed through the appropriate rotation or reflection at each stage with this rebuilding step.
The upper left hand cell is rotated 180°.
The lower left hand cell is reflected about a vertical axis.
The lower right hand cell remains in it original orientation.



Look for the characteristics of self-similarity. Three reduced images of stage 2 are in stage 3. Three reduced images of stage 1 are in stage 2, and three reduced images of stage 0 are in stage 1.

Resource Book

Workshop 10: Generating Fractals

Workshop Outline

1. **Being precise**
   a. We paired up, then took turns describing the figures on the overhead projector to our partner. This activity helped us to see the value of precision.
   b. We saw that these figures could be described by
      i. describing a starting stage, and
      ii. describing an iterative rule that worked at every stage.

2. **Reduce, Replicate and Rebuild**
   a. We saw that the Sierpinski triangle of yesterday could be described another way (in addition to that in which we removed middle quarters of the triangles) Namely, by reducing, replicating three times and rebuilding.
   b. We then tried this in a square — reducing, making some copies and rebuilding:
      i. with 1 copy placed in a corner, we converged just to a single point, a corner of the square.
      ii. with 2 copies, we converged to a line segment; a side of the square or a diagonal.
      iii. with 4 copies, we always retained the full square.
      iv. but with 3 copies, we generated a Sierpinski-like structure, rather surprisingly.
   c. We saw, therefore, that it was not as important what initial shape you started with as what your rebuilding rule was. We could even start with mailboxes and generate a Sierpinski-like structure.

3. **Introducing rotations and building a wallpaper fractal**
   a. We then rotated two of our copies, and saw that a radically different structure emerged.
   b. After building a few stages on the overhead, we were each handed 9 copies of stage 6 of this particular fractal, and together built the wallpaper fractal, an activity designed by Evan Maletsky.

4. **Generating our Very Own Fractal**
   a. By choosing how we would rotate each of the three quadrants of the square that we used (all but the upper right, by convention) we had 4×4×4 different possibilities, yielding 64 different fractals we could create. We would never get bored.
   b. We then saw that there were 16 possible rules if we restricted the lower left quadrant to be unrotated.
   c. Each group was assigned one of these 16 rules, M, I, N or K, and each group generated their Very Own Fractal.
   d. We then tried to match rules with the stage fives that they generated.

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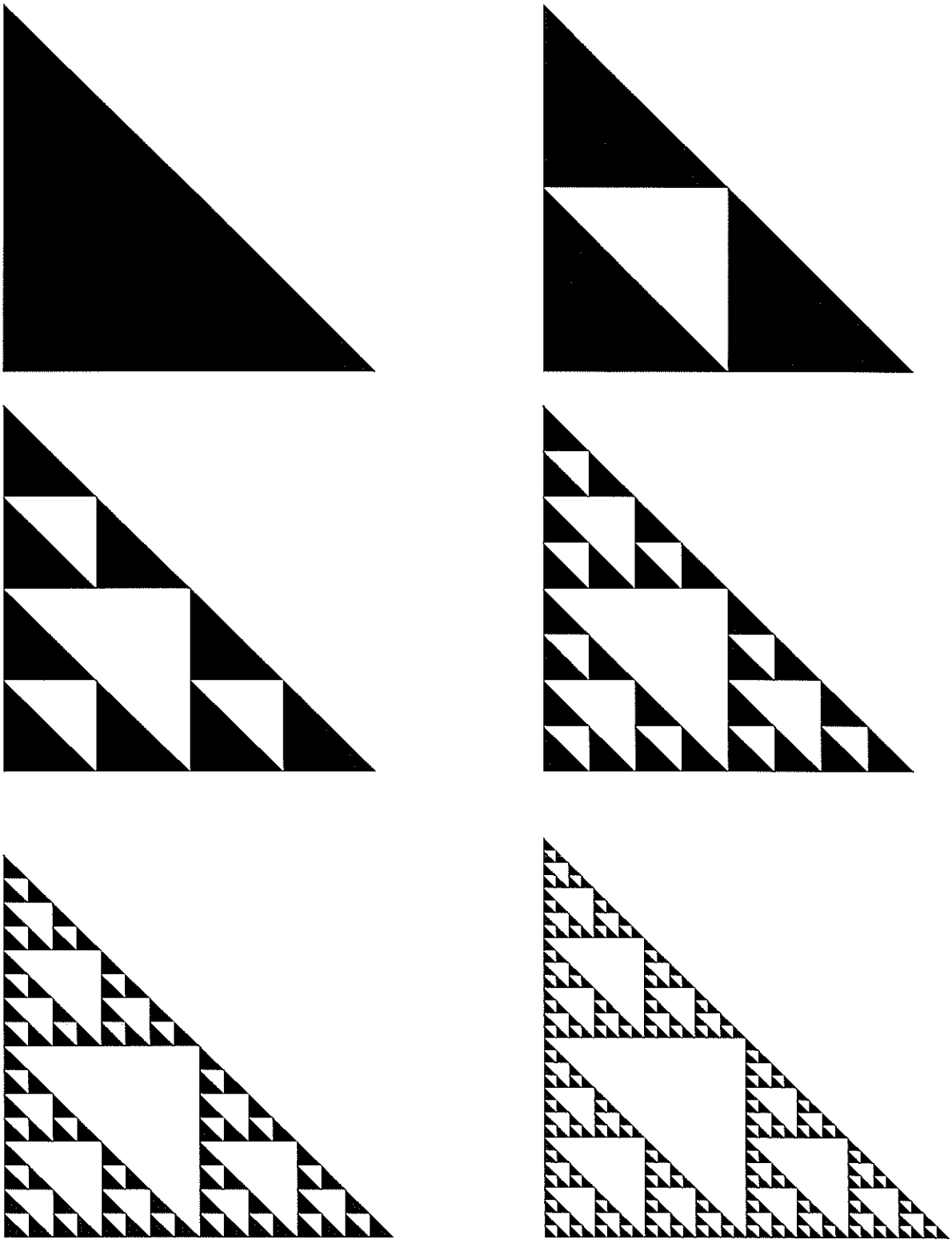
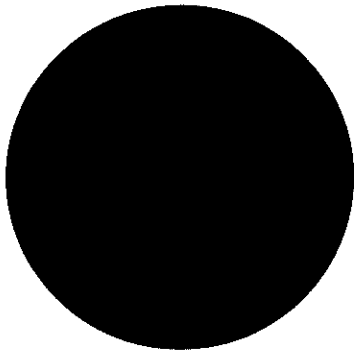
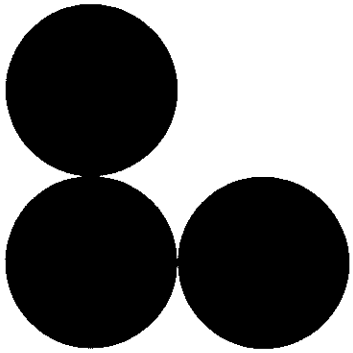
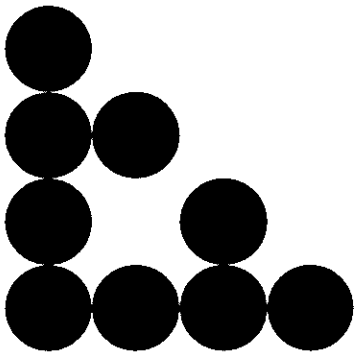
Workshop 10: Generating Fractals

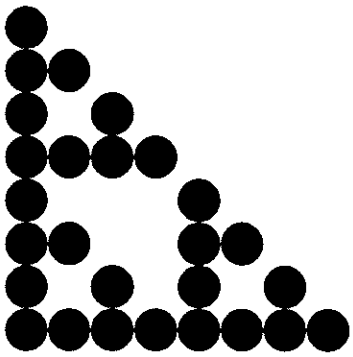


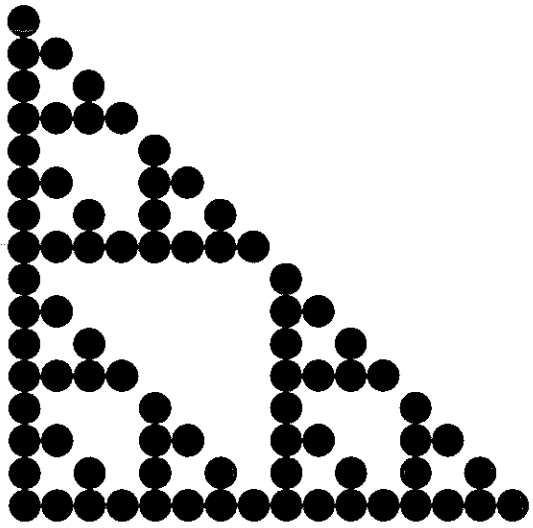
Workshop 10: Generating Fractals

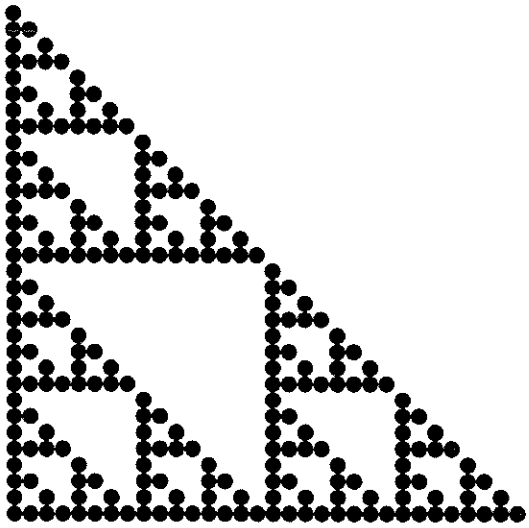




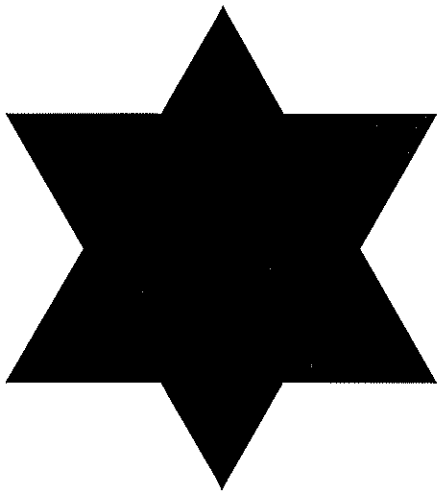


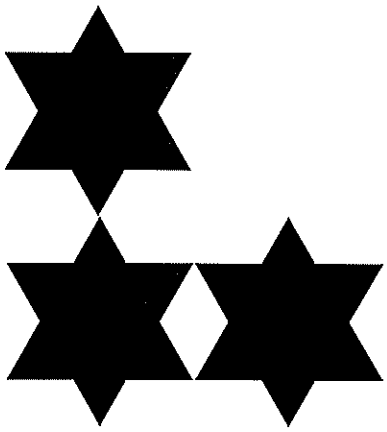


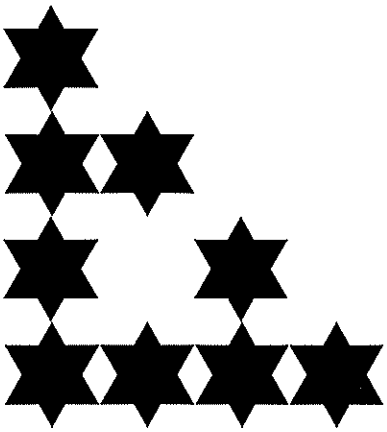


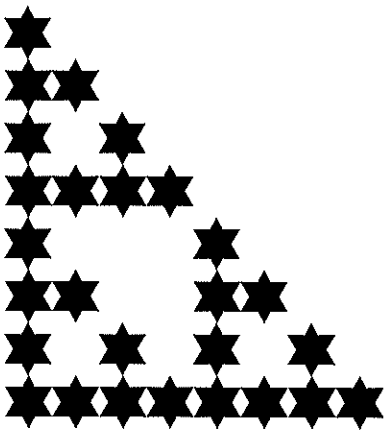


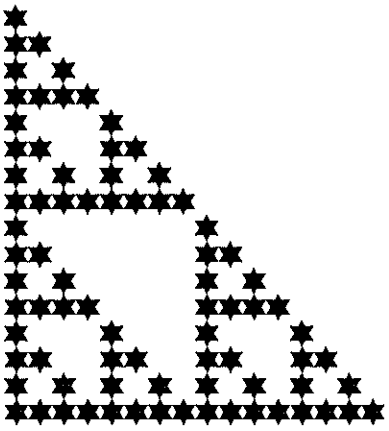
Workshop 10: Generating Fractals

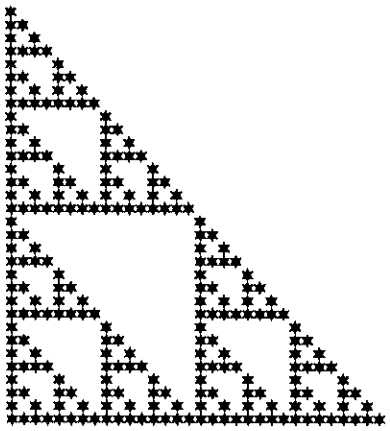




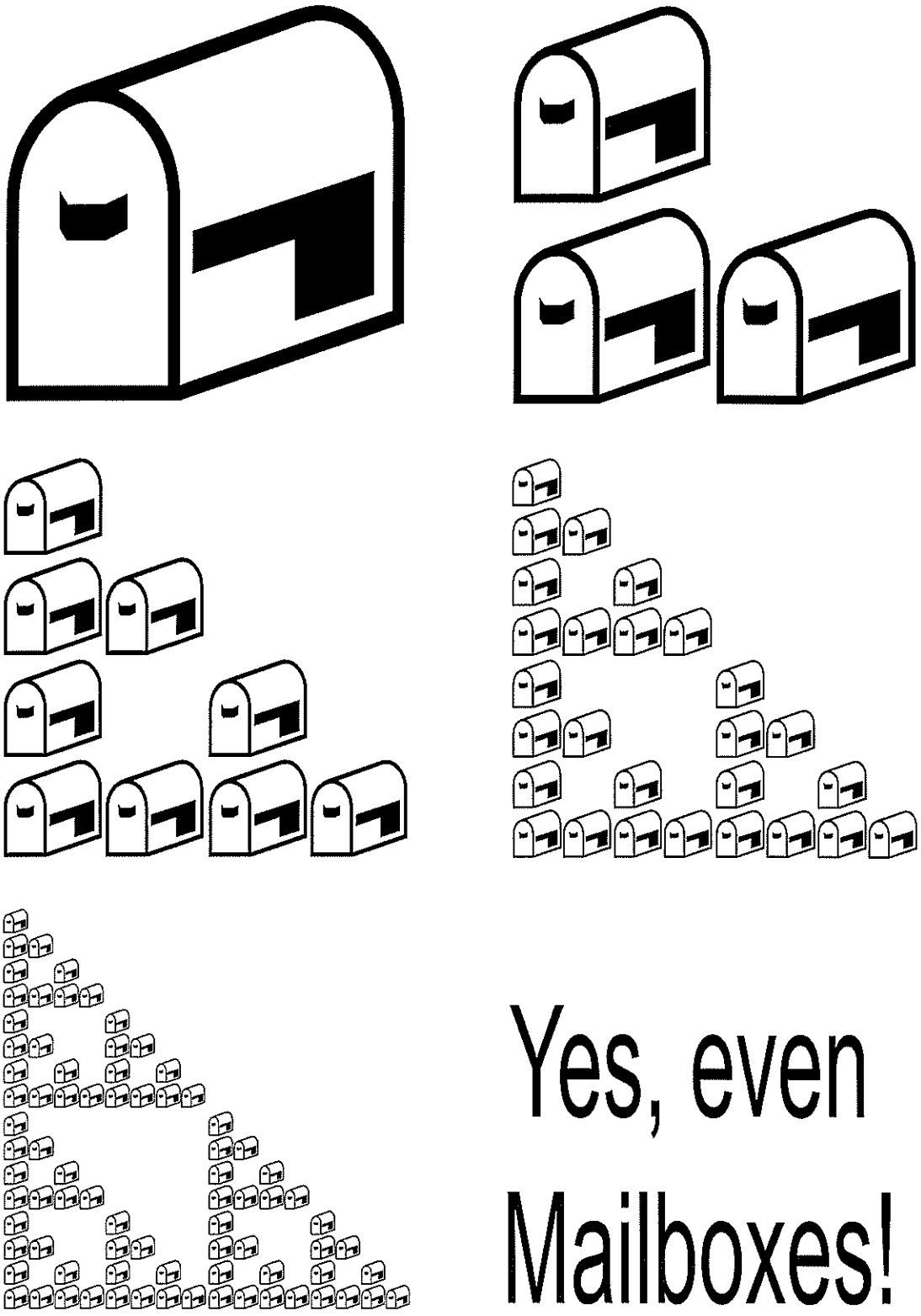




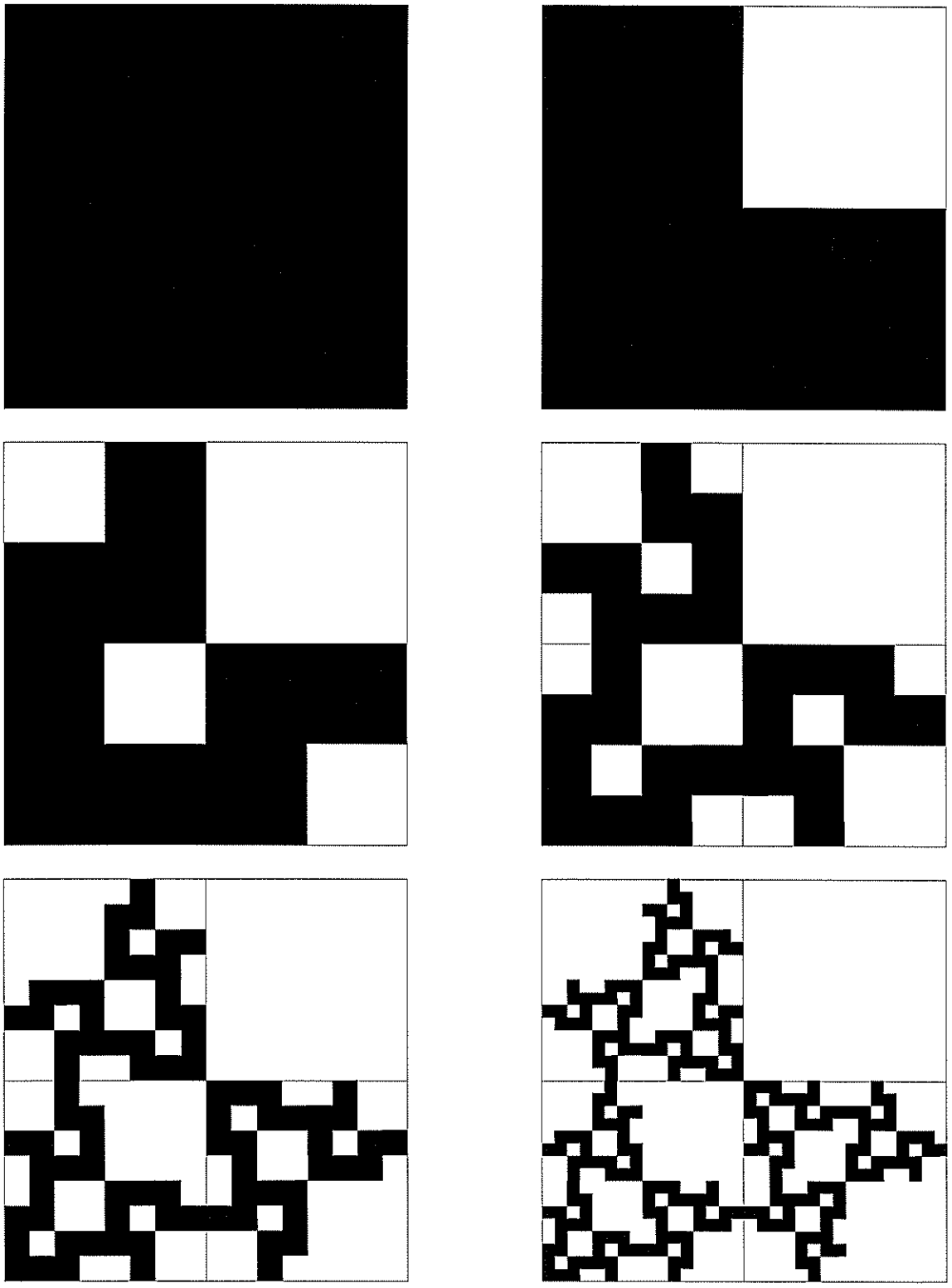




Workshop 10: Generating Fractals



Workshop 10: Generating Fractals



Resource Book

Workshop 10: Generating Fractals

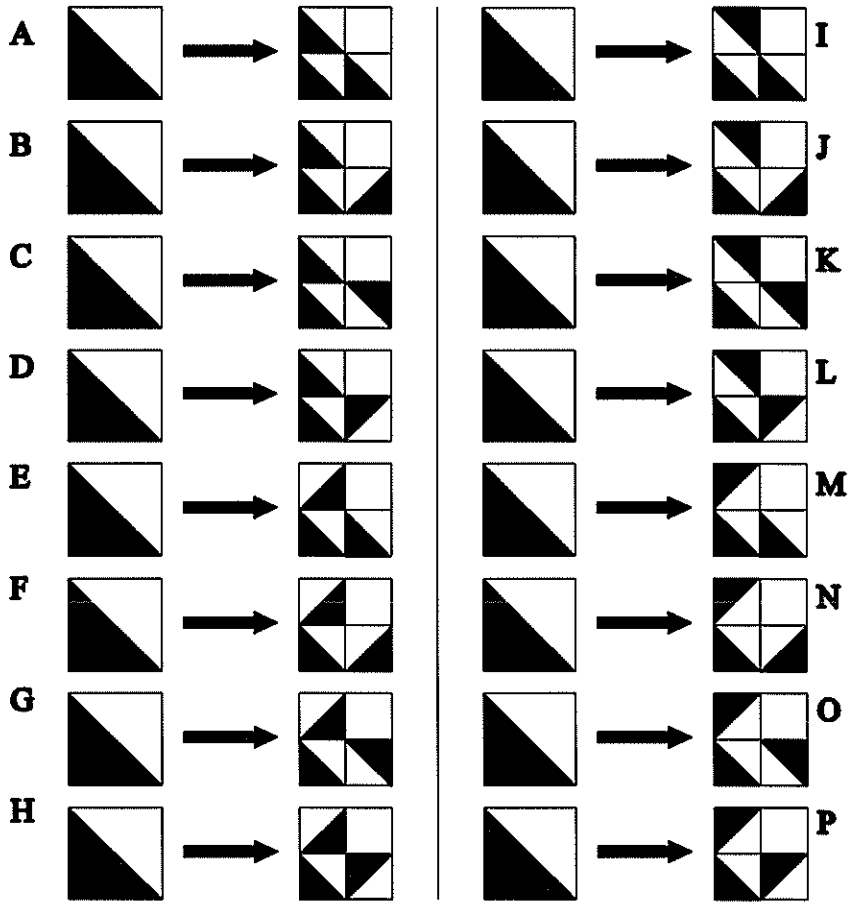
Variations for the rebuild step

If we generate a fractal by starting with a right triangle (so that we can see the rotations), and at each stage:

- Reduce by 50%
- Replicate 3 times, and
- Rebuild by placing an unrotated copy in the lower left quadrant, and a rotated copy in each of the lower right and upper left quadrants

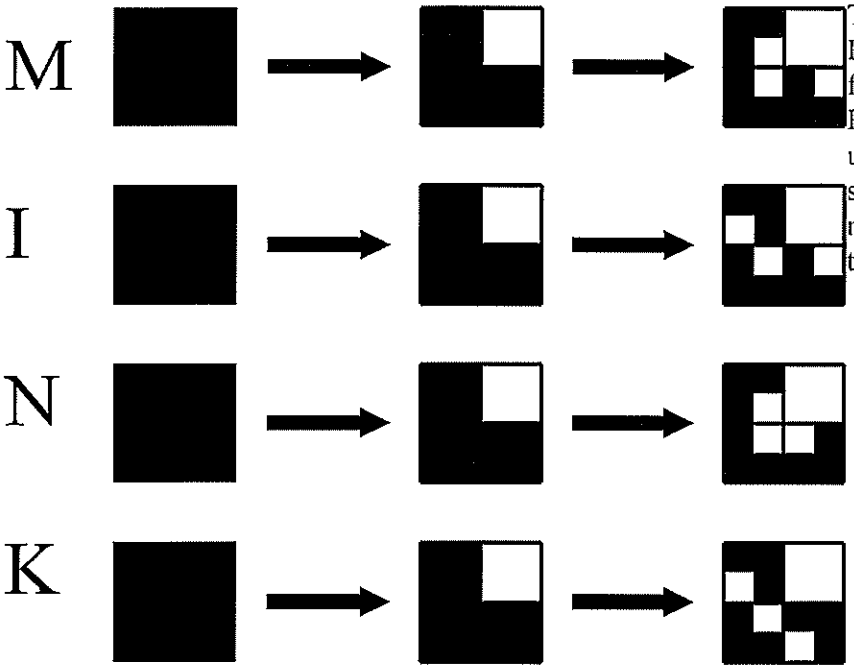
then there are 16 possibile rebuilding rules. These are all shown below.

Note that "A" yields the Sierpinski triangle, and "H" yields the one we built during workshop.

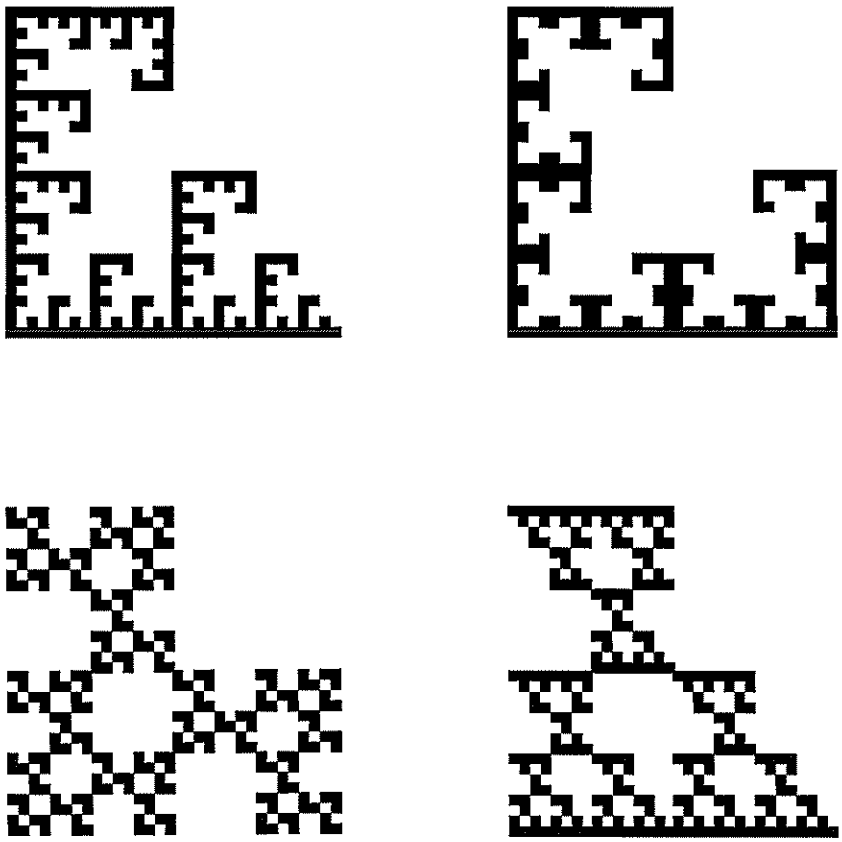


Rot	Blank
Orig	Rot

Workshop 10: Generating Fractals



These four rules, taken from M, I, N and K in the previous page, generate the four fractals (at stage 5) shown below. Proceeding clockwise, starting from the upper left, we have M, N, I and K. Two stages are shown since one stage would not have revealed the rotations that were taking place.



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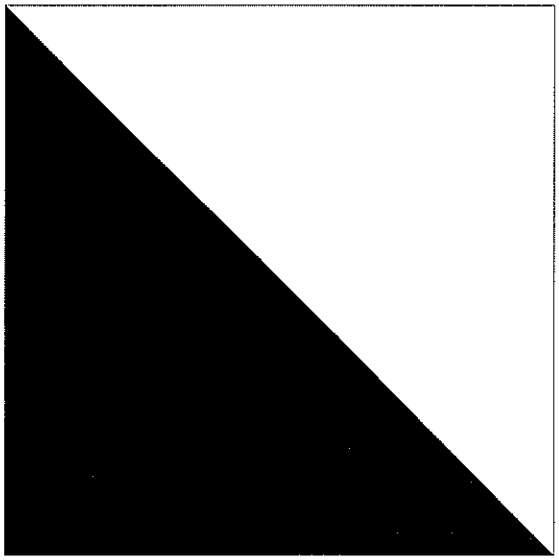
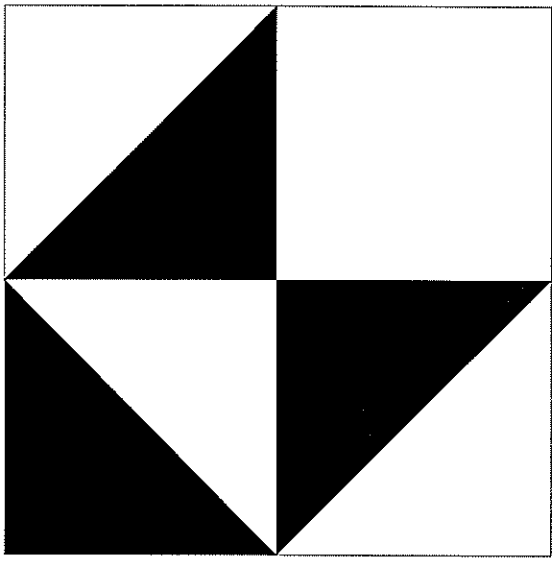
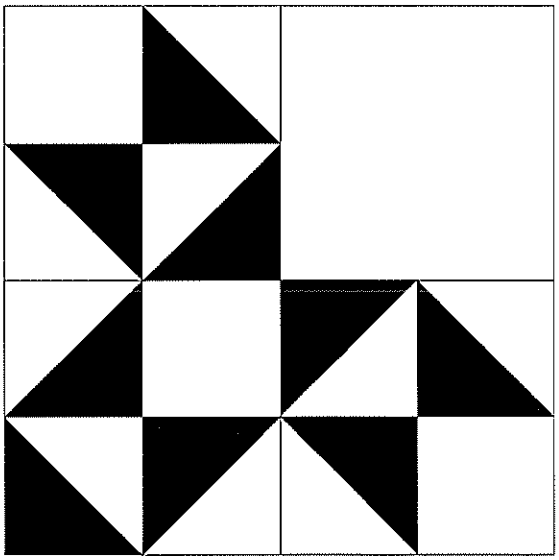
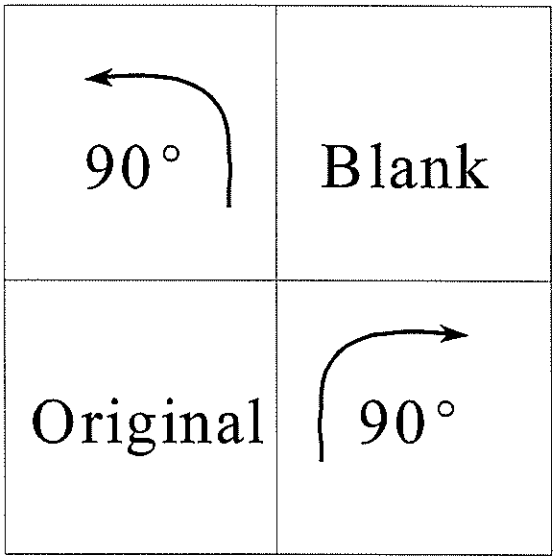








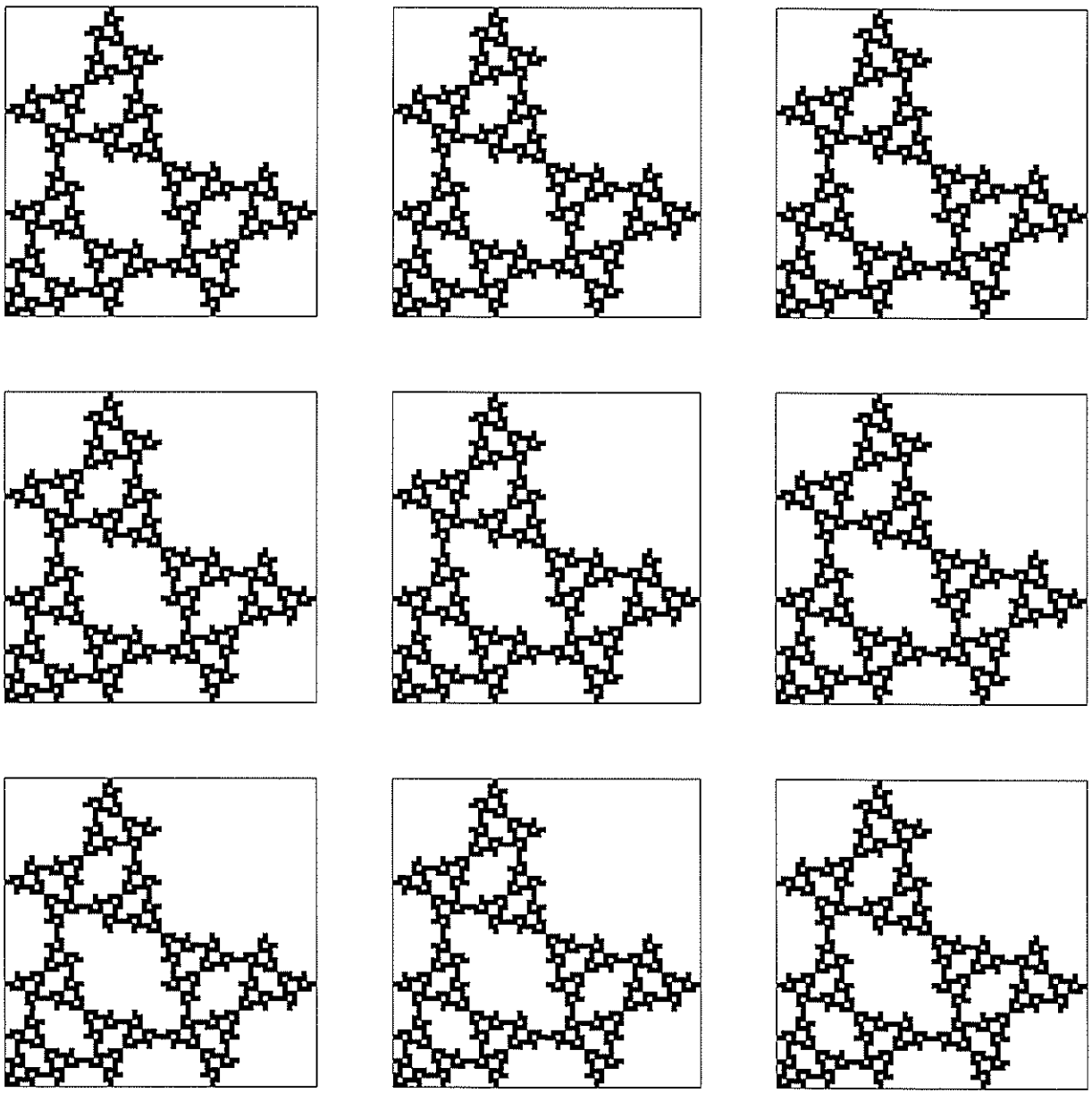
Iterating Instructions

Resource Book

Workshop 10: Generating Fractals

FRACTAL GENERATION
Repeated Copies

Dr. Evan Maletsky
Montclair State University



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Workshop 10: Generating Fractals

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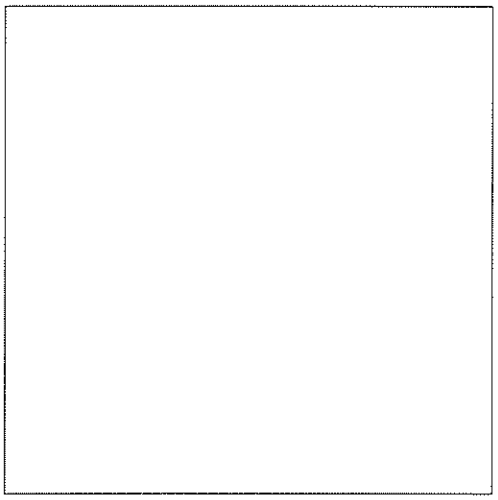
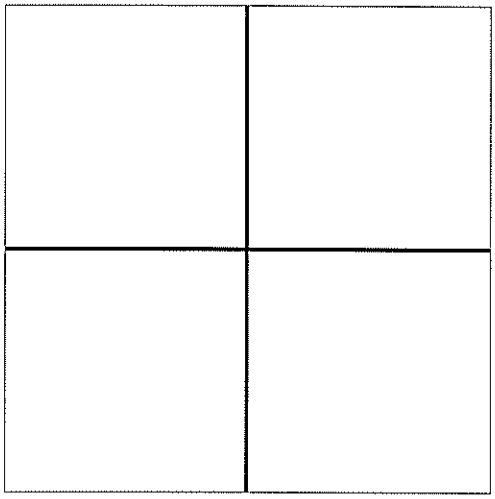
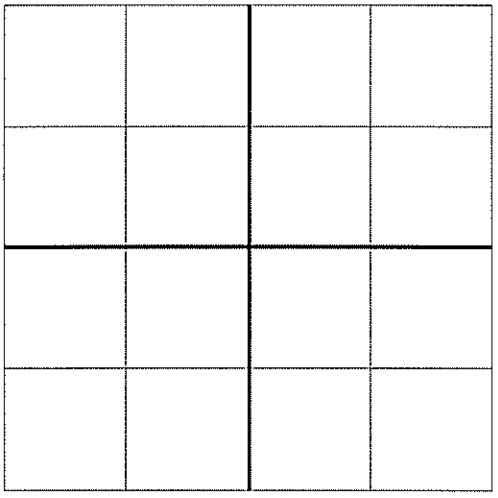
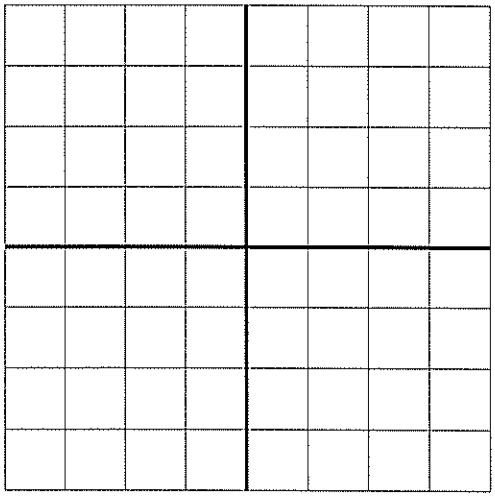
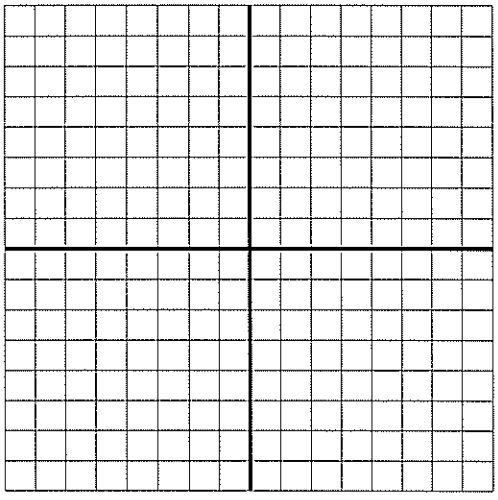
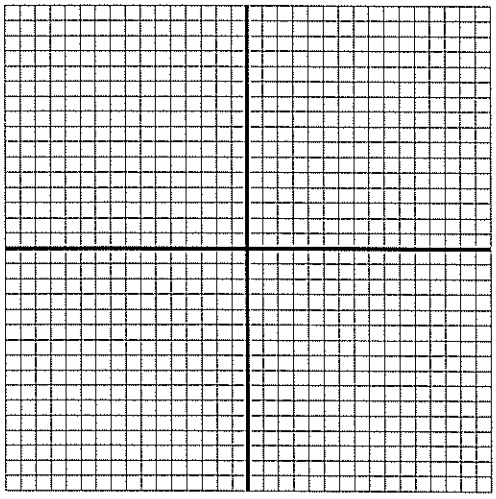








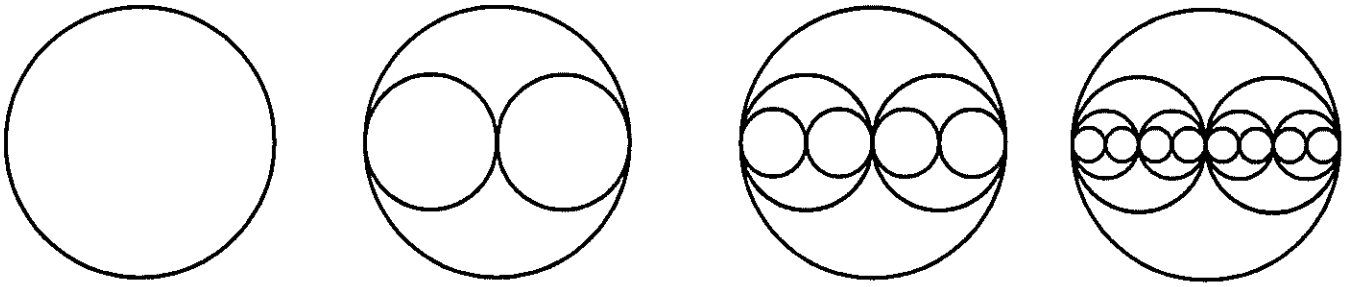


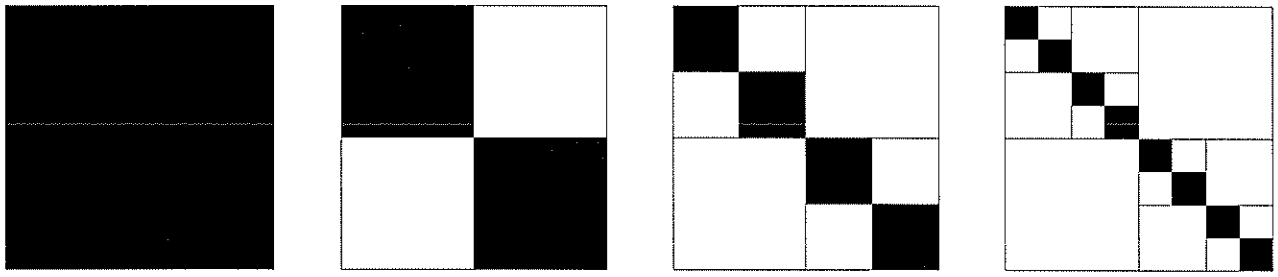


GRIDS FOR 2X2 REPLICATION

Workshop 10: Generating Fractals

WRITE DOWN A FEW SENTENCES TO DESCRIBE THESE TO A PARTNER.

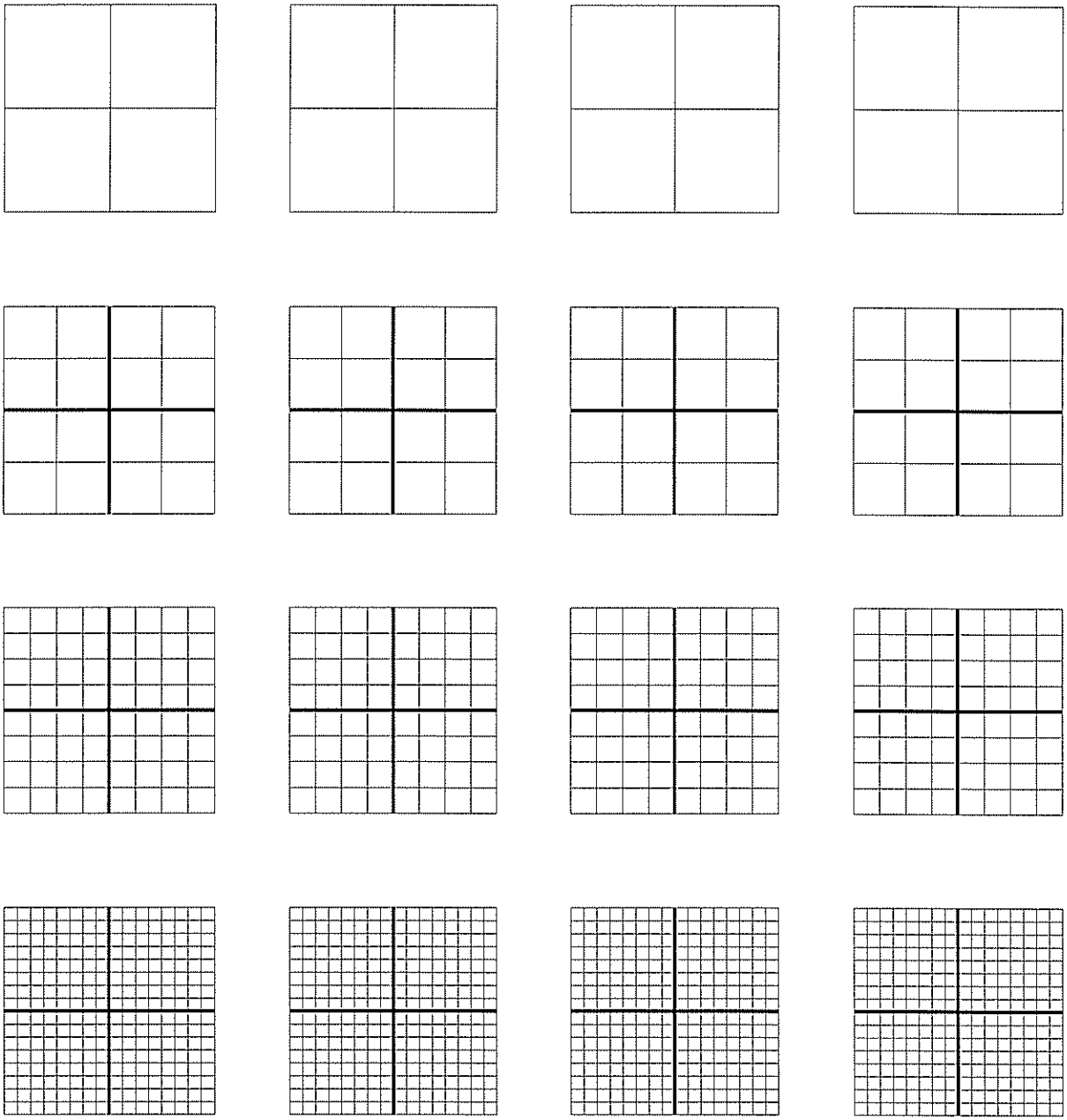




Resource Book

Workshop 10: Generating Fractals

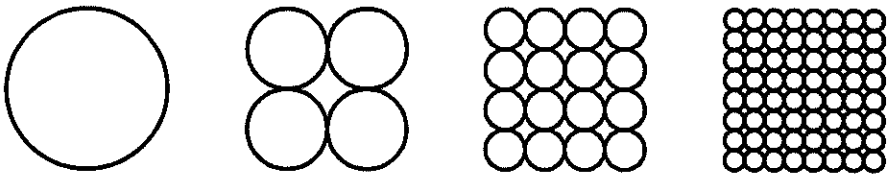
Grids for making fractals in 2x2 squares.



Resource Book

Workshop 10: Generating Fractals

REDUCE, REPLICATE AND REBUILD



REDUCE: _____

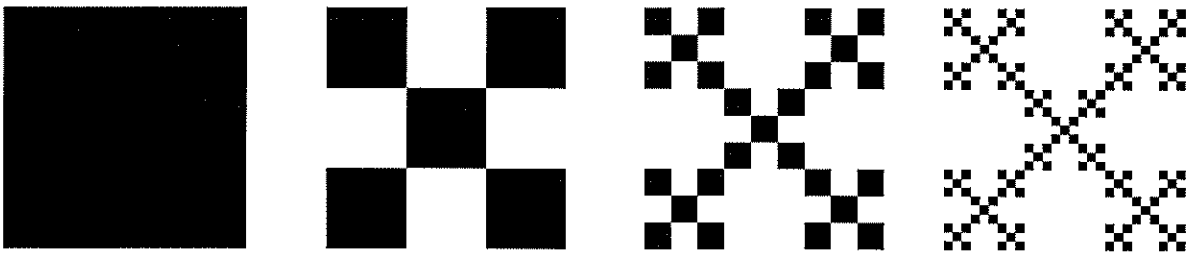
REPLICATE: _____ TIMES

REBUILD: _____

_____

_____

_____



REDUCE: _____

REPLICATE: _____ TIMES

REBUILD: _____

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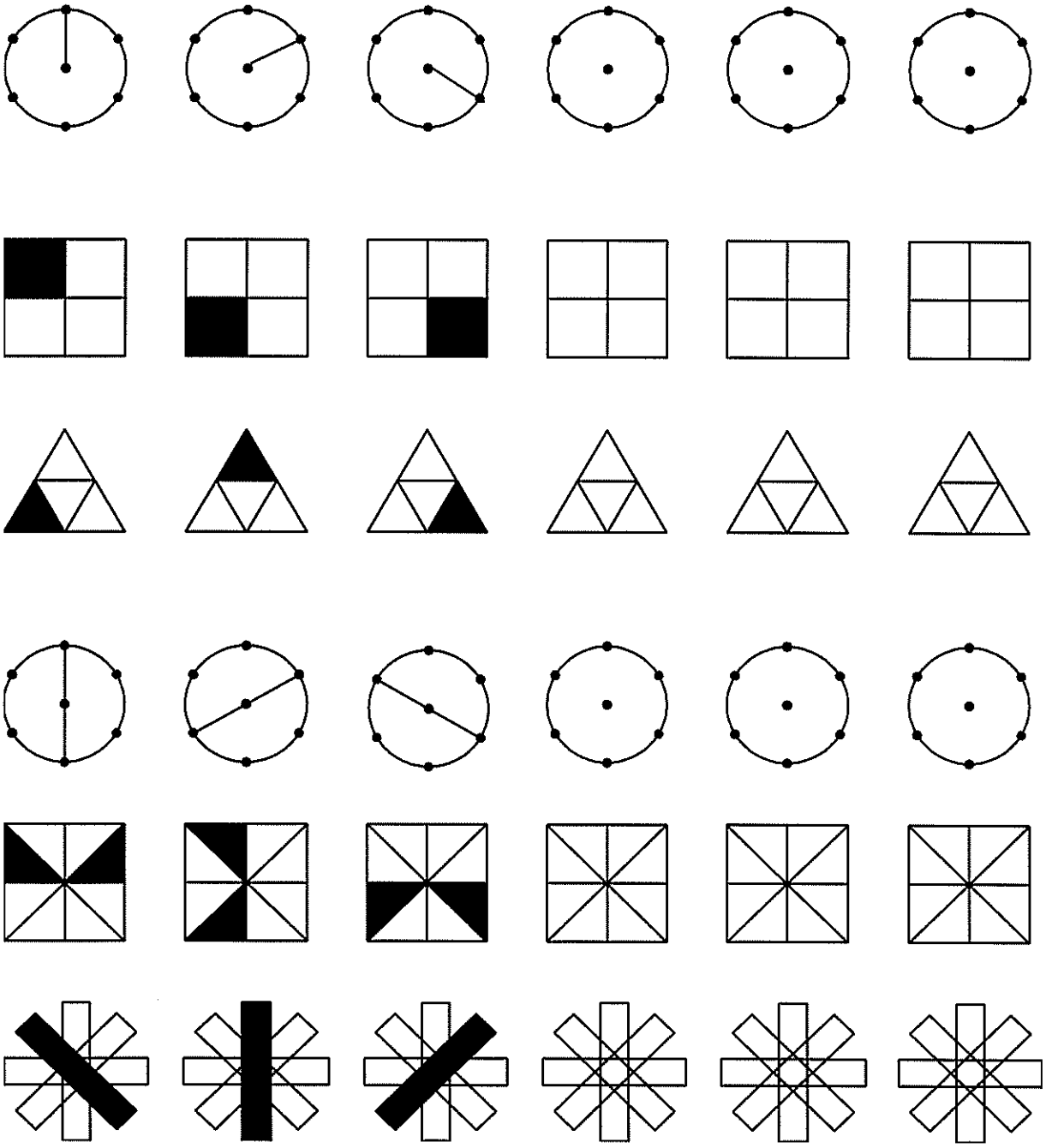
Workshop 10: Generating Fractals

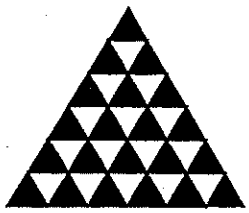
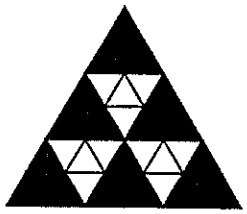
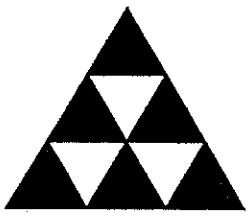
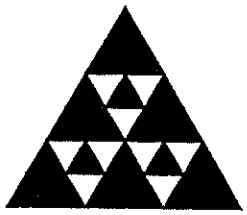
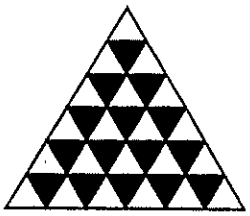
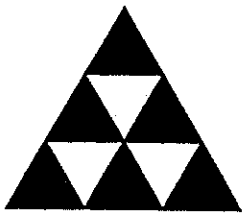
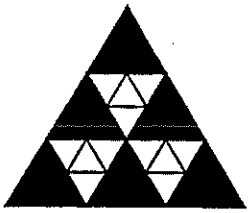
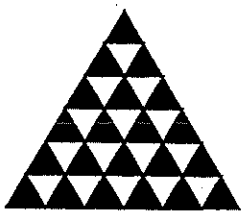
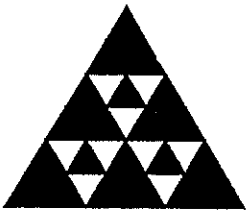
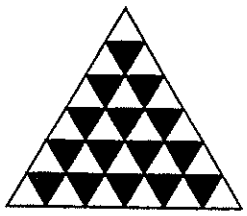
CONTINUE THE PATTERN **Rotations**



By Robert Hochberg. Copy as you wish.

TRIANGLE MATCH

Draw a line between each of the triangles that match.

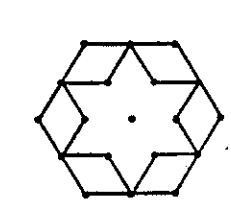











Workshop 10: Generating Fractals

GRID DESIGNS

★★★

Copy each design on the blank grid below.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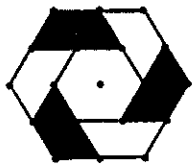
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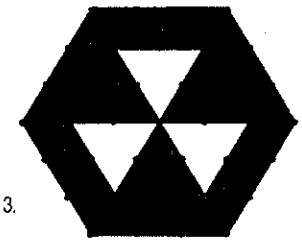
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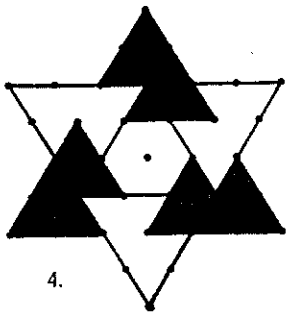
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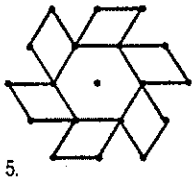
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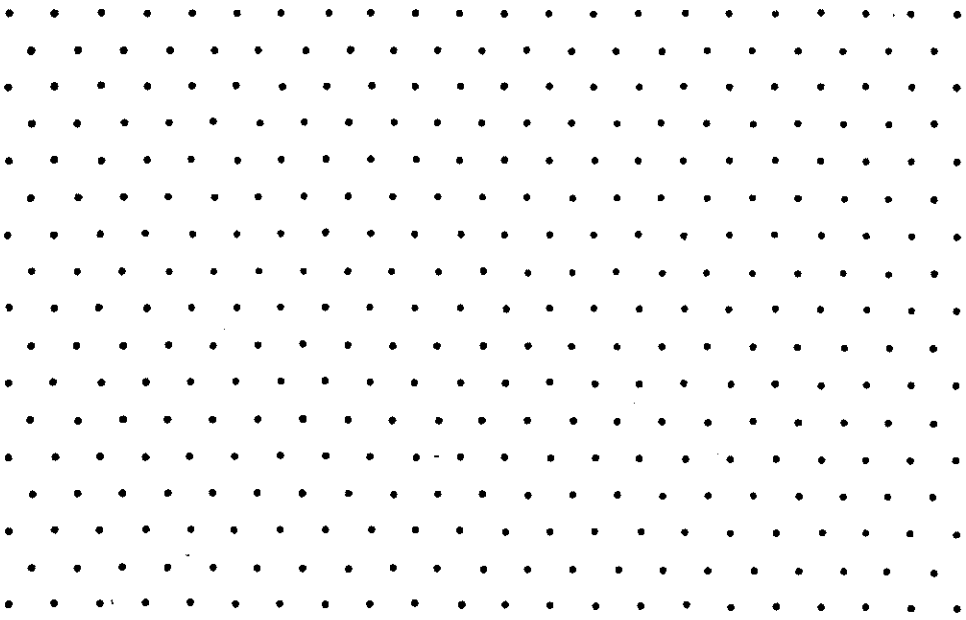
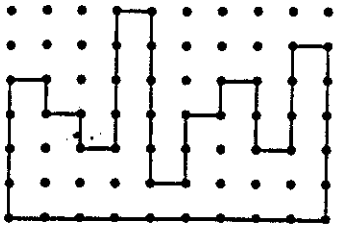
6.

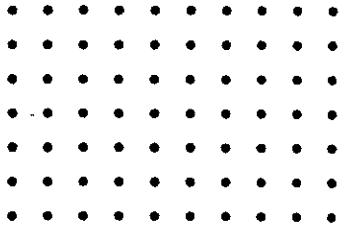


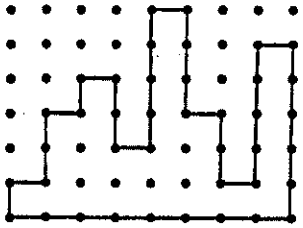
Workshop 10: Generating Fractals

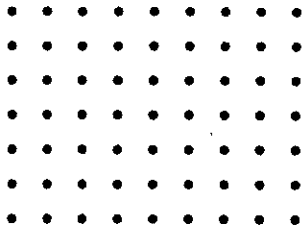
DOT DESIGN (I)

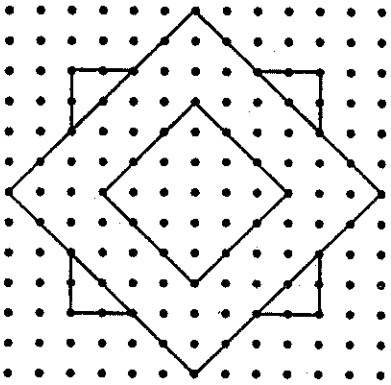
Copy each design.

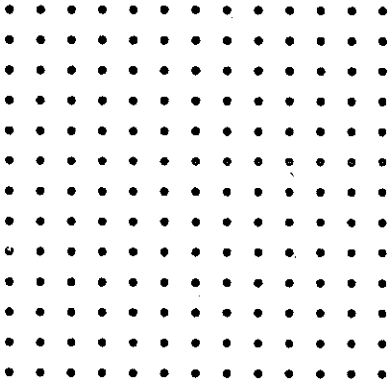








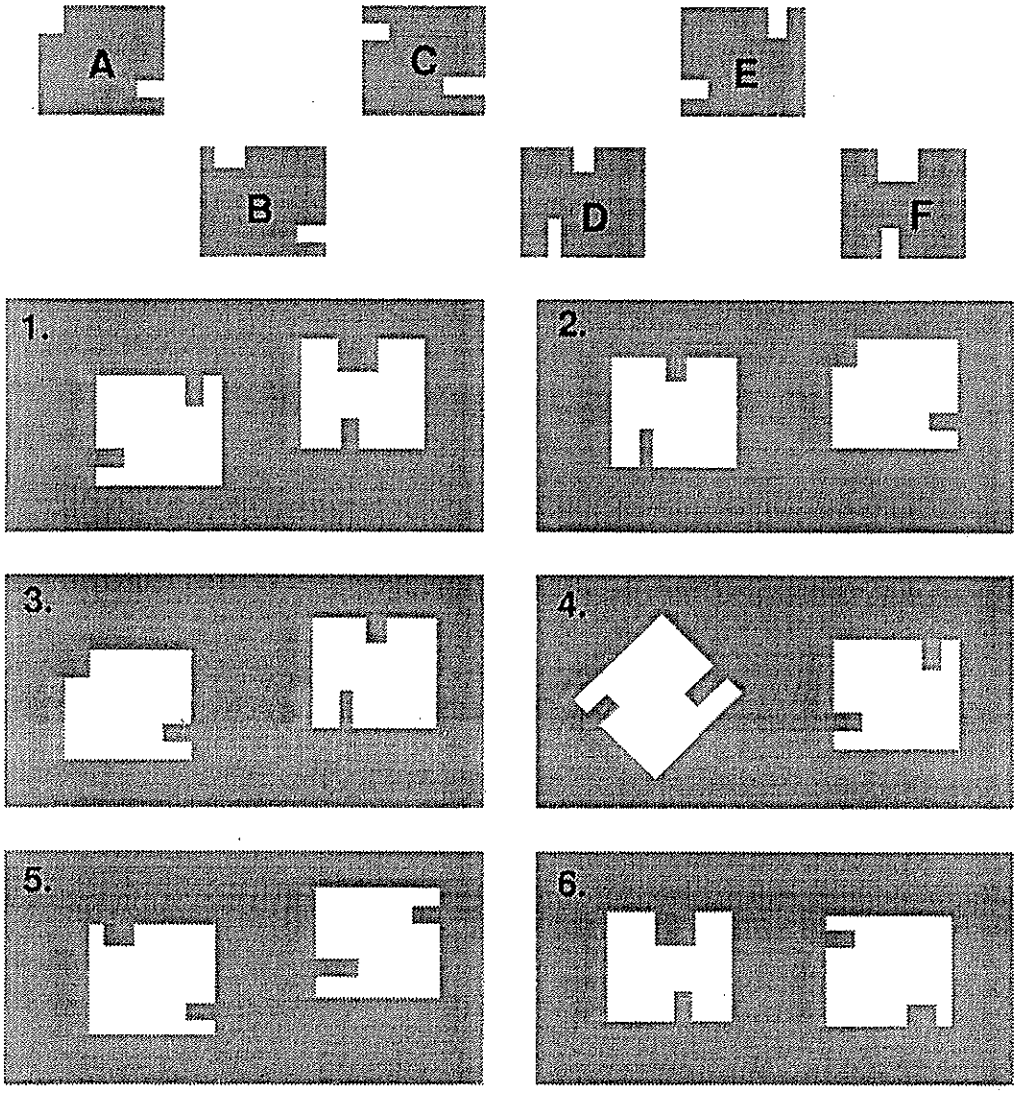




MISSING PIECE (III)

▼▼▼

Write the letter of the missing pieces in each of the six problems.

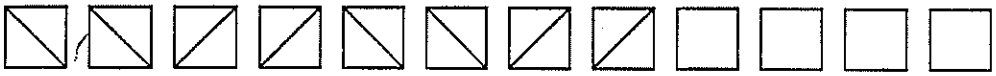


DRAW THE PATTERNS (III)

Continue each pattern.

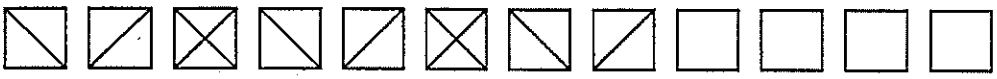
1. 

2. 

3. 

4. 

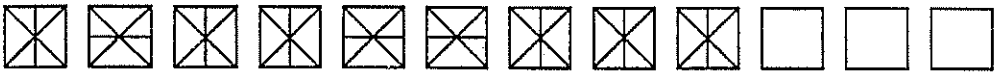
5. 

6. 

7. 

8. 

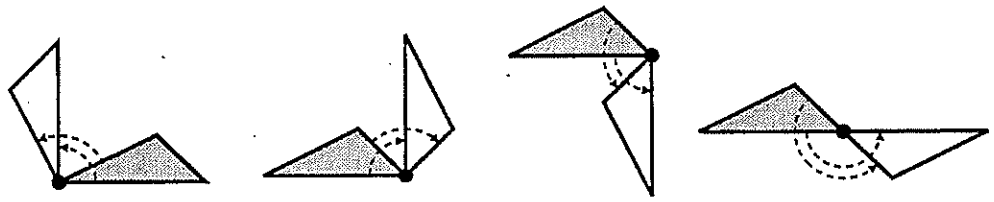
9. 

10. 

Workshop 10: Generating Fractals

IT'S YOUR TURN ◆◆◆

Rotation can be either clockwise or counterclockwise—and around any point in the plane. The examples below show grey triangles rotated in a plane to the position of the white triangle.



Example 1
Rotate 90° counterclockwise(CCW)

Example 2
Rotate 90° clockwise (CW)

Example 3
Rotate 90° counterclockwise (CCW)

Example 4
Rotate 180° counterclockwise (CCW)

Draw the figure on the grid rotated, around the dot, as indicated: