Increasing the Quantity and Quality of the Mathematical Sciences Workforce Through Vertical Integration and Cultural Change

Stories of Innovations and Culture Change

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September 2008

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CHAPTER 1
INTRODUCTION

"Science (including mathematics) and technology have been and will continue to be the engines of U. S. economic growth and national security. Excellence in discovery and innovation in science and engineering derive from an ample and well educated workforce." This statement, which appears in the National Science Foundation Board Report: *The Science and Engineering Workforce – Realizing America's Potential,* declares that a driving necessity is a robust science, mathematical sciences, and technology workforce for the 21st century.

> **"Human resources are the most critical element to innovation and economic prosperity for companies and countries alike," stated W. J. Sanders III, founder and Chairman Emeritus of Advanced Micro Devices, Inc. at the *Innovation Initiative Summit* in Washington, D. C. in December 2004.**

For centuries the centrality of the mathematical sciences was clearly understood. Carl Friedrich Gauss referred to mathematics as the *Queen of the Sciences*. Roger Bacon in the 13th century declared that "Mathematics is the door and key to the Sciences." Today, there is an increasing need for mathematical sciences in all fields of science, engineering, health, and the social sciences, yet there is appears to be a looming critical shortage of U. S. students and faculty in the mathematical sciences. The *2006 National Science Foundation Science and Engineering Indicators Report* predicts a 39% increase in employment opportunities for mathematicians and computer scientists, more than the 26% increase in employment for scientists and engineers (S&E) more generally (includes the math/CS people), between 2002 and 2012, and significantly more than the projected increase of 15% for all jobs. See Figure 1.

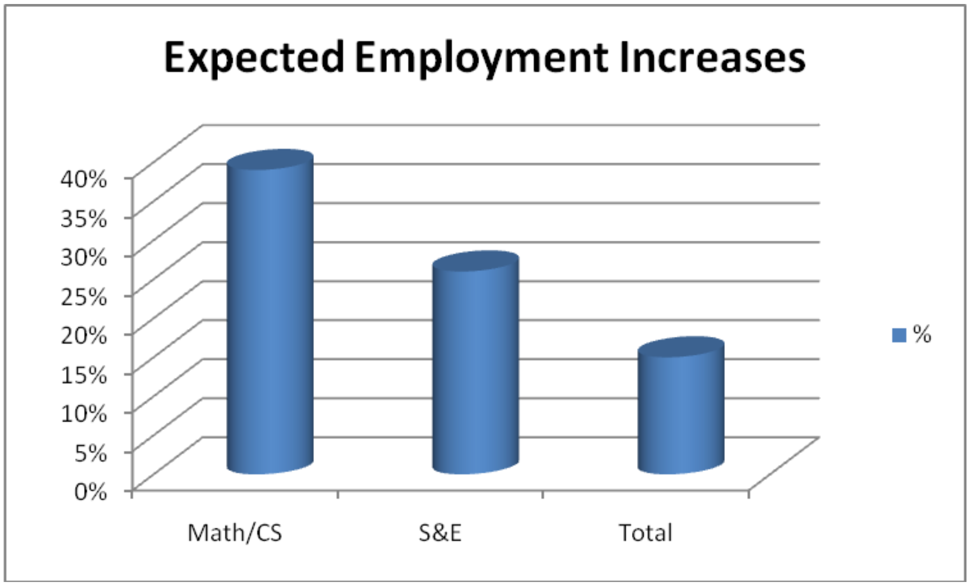

Figure 1: Expected employment increases from 2002 to 2012.

Source: *National Science Foundation Science and Engineering Indicators 2006* Table 3-2

Numerous factors have come together to contribute to these projected shortages, including the retirement of baby-boomers, the graying of the workforce, the challenges of attracting a diverse population of U. S. students to the mathematical sciences, and the increasing demand for mathematical sciences expertise in almost every sector of the economy.

The National Academy of Sciences, the National Academy of Engineering, and the Institute of Medicine in their document *Rising Above the Gathering Storm – Energizing and Employing America for a Brighter Economic Future*, commissioned by the United States Congress, points out that the scientific and technological building blocks critical to our economic leadership are eroding at a time when many other nations are gathering strength (3). They found that it is critical to optimize knowledge-based resources, and they recommend making the United States the most attractive setting in which to study and perform research so that we can develop, recruit, and retain the best and the brightest students, scientists, and engineers from within the United States and around the world.

Thomas Friedman in his book, *The World is Flat*, talks of various gaps between the United States and other countries, which impede the U.S. ability to be economically successful: a numbers gap, an ambition gap, and an education gap. ---"We need to do things differently as we did when we put a man on the moon. There are no shortcuts; it all takes time." Friedman, speaking of graduate education in this country, says that this country needs to look forward, not backward, that "we can mine the present to produce the future." (2)

The Division of Mathematical Sciences at the National Science Foundation, recognizing the need to address critical workforce issues in the mathematical sciences in the United States in the 21st century and to fill the numbers gap, in 1999, established the **Vertical Integration of Research and Education in the Mathematical Sciences (VIGRE) Program**. The goals of projects funded as part of this program were and still are to:

- integrate research and education;
- enhance interaction across undergraduates, graduates, post-doctoral fellows, and faculty;
- broaden the educational experiences of students to include workforce and early research opportunities; and
- motivate more students, especially women and minorities, to study mathematics and statistics.

Mathematical sciences departments in the VIGRE program, and in this book, include departments of Applied Mathematics, Mathematics, and Statistics. The Appendix lists all of the universities and their departments receiving awards under the VIGRE program.

Chapter 2 of this book looks at these specific VIGRE goals and provides examples of how various VIGRE projects addressed these goals successfully. Not all VIGRE projects were successful in addressing all four goals, but most of the projects had very successful components that addressed some of the goals. The stories in this section describe some of the ways different universities implemented new or revised practices to address the four goals of VIGRE.

Chapter 3 considers some of the major ingredients of transforming mathematical sciences departments into dynamic research and education places: leadership, strong mentoring, dynamic curriculum, high quality research and access to it, and availability of internships. The stories in this chapter provide insight into creative ways that departments have built on what they had in place already and added to it.

Chapter 4 indicates that VIGRE programs have gone beyond the original goals in order to extend their reach and potential for sustainability. These universities learned early on that in order to be successful they needed to expand the pipeline at the K-12 level and that meant working with high schools in their area. They discovered that by partnering with other departments and other universities, they became stronger and helped to create a more dynamic mathematical sciences workforce.

Chapter 5 provides data to suggest that quality education coupled with quality research does make a difference, and is translated into increased numbers of U. S. students entering and completing undergraduate and graduate degrees in the mathematical sciences. Both the quantity and the quality of students in many of these programs have increased over the eight years of VIGRE. More women, and more minority students are entering the mathematical sciences as they recognize that they are welcome, talented, and supported.

The concluding chapter, Chapter 6, includes a specific section on suggestions for other mathematical sciences departments, those with graduate programs and those without them, to respond to the 21st century demands for mathematical scientists at every level. This chapter also includes a section on sustainability, what is being sustained after VIGRE funds end, what is not, and what is possible long term, and a final section on effective practices.

A working title for the book has always been *success stories out of VIGRE*, but to use that title one needs to define success in the context of VIGRE. Success for VIGRE has come to mean successful innovations, with results which lead to a collegial environment where both research and education thrive for students and faculty. The firmly held belief, substantiated by some data, is that if a collegial environment for research and education exists, then students will be drawn to the mathematical sciences, especially U.S. students. The chapters of this book illustrate that innovation has indeed occurred and that positive results have been realized in many ways, some small and some large, at over two-thirds of the VIGRE sites. There is now a cache of innovations available to these and other mathematical sciences departments to broaden the influence of VIGRE.

This book is **not** intended as an objective evaluation of VIGRE – it does not include what happened at unsuccessful sites, nor does it draw any comparisons with mathematical sciences departments who did not receive VIGRE awards - some may have made positive changes in integrating research and education on their own. VIGRE set a context for and provided funding to encourage change, but no attempt is made in this book to rigorously prove that any specific change was a direct result of VIGRE. The VIGRE mathematical sciences departments strongly believe that significant changes are attributable to the VIGRE program, however, and that there has been considerable value-added from the VIGRE program. The National Academy of Sciences at the request of NSF is conducting a formal evaluation of the VIGRE program and will issue its report in late 2008.

This book **is** intended to provide a set of examples, told as stories, of things that can be done in mathematical sciences departments across the country to improve the quality of mathematical sciences education, not only to address critical workforce issues in the U S, but to just make things better and foster good working relationships. The stories told are chosen to provide examples of activities and changes made by mathematical sciences departments funded by the VIGRE program. It is by no means exhaustive of all activities, or of all of the activities of any one department. These stories provide a compendium of best practices for use by other mathematical sciences departments.

The material for this book was gathered from written reports, websites, conversations, and visits to a number of VIGRE sites. The book is much improved thanks to careful reviews by Helen Tuffel, APEX Analytix and a technical editor, and the Advisory Board for the project:

- Dr. Mel George, Mathematician and former President of the University of Missouri and St. Olaf's College
- Dr. Karen Kafadar, Professor of Statistics, Indiana University
- Dr. Joan Leitzel, Mathematician and former President of the University of New Hampshire
- Dr. David Manderscheid, Professor of Mathematics and Dean of the College of Arts and Sciences at the University of Nebraska at Lincoln
- Dr. Judith Ramaley, Professor of Biology and President of Winona State University
- Dr. David Roselle, Mathematician and former President of the University of Delaware
- Dr. David Sanchez, Mathematician and former Provost at Texas A&M University

The author is fully responsible for any inaccuracies in the book, and all opinions expressed in the book.

CHAPTER 2

Vertical Integration of Research and Education in the Mathematical Sciences

The Division of Mathematical Sciences at the National Science Foundation in 1999, responded to reports of the declining numbers of undergraduate and graduate students in the mathematical sciences, and they established the **Vertical Integration of Research and Education in the Mathematical Sciences (VIGRE)** program to prepare an increasing number of U. S. citizens for careers in the mathematical sciences. Mathematical Sciences departments (Applied Mathematics, Mathematics, and Statistics) in universities across the country applied for substantial grants under the VIGRE program. In all, 39 universities, which are listed in the appendix, received these grants. There are a number of elements common to all of the projects, and a number of elements unique to each one. Each project endeavored to

- integrate research and education;
- enhance interaction across undergraduates, graduates, post-doctoral fellows, and faculty;
- broaden the educational experiences of students to include workforce and early research opportunities; and
- motivate more students, especially women and minorities, to study mathematics and statistics.

These four aspects of all projects are accomplished through a strong commitment to educating students in the mathematical sciences, those who become research mathematicians, those who go on to teach at the K-12 level, those who go on to work for the government, and those who find careers in industry. Everyone is responsible for this education - students, faculty, and the community. Everyone facilitates the transitions from high school to college, from undergraduate to graduate school, from graduate school to postdoctoral fellow, faculty or the workplace. These transitions have other transitions explicit within them, for example the transition from course work to original research and the transition from postdoctoral fellow to tenure track faculty. People change as a result of what they notice, not just what they are told, and students and faculty have a much better chance to make observations in a vertically integrated environment.

2.1 Integration of research and education

Integration of research and education occurs at many levels and across many levels at many VIGRE sites. At the University of Chicago, mathematics undergraduates who participate in the summer research experience programs are mentored by graduate students and faculty, and they in turn work with and help educate high school students in the Young Scholars Program or work with middle school teachers in the SESAME Program.

University of Chicago REU

The Summer Research Experiences for Undergraduates (REU) Program at the University of Chicago is an eight week program for junior and senior mathematics majors, and a four week summer program for freshman and sophomore mathematics majors (called the Apprenticeship Program). The Chicago REU program began the first year of the VIGRE grant in 2000 with 18 students and in 2007 had 80 students. In this program students have the opportunity for study and research in mathematics and for work with two outreach programs of the Department of Mathematics. This VIGRE REU program provides students with involvement in a deeper experience in mathematics than is usually available during the academic quarter. It is especially beneficial for undergraduates who are considering graduate study and research in mathematics and for those who are interested in teaching at any level.

REU students participate in at least one of four courses taught by faculty members. The courses consist of lectures by faculty and problem solving sessions run by graduate students. Research problems and some problems aimed simply as an aid to understanding mathematics are introduced by the professors. No previous knowledge or study in the areas taught is required. Students in the four-week apprenticeship program have a schedule similar to the eight week program, but the mathematical material is at a level requiring less mathematical experience. Students often participate as an apprentice one summer and then in the full program the following summers. Graduate students are on hand to help both groups of students. Each REU student is paired with a specific graduate student mentor, whom they meet with on a regular basis.

Each VIGRE REU student serves as a counselor for either the four-week 7^{th}-12^{th} grade students in the Young Scholars Program, or the two week program for elementary school teachers in the Chicago Public Schools, called SESAME. Counselors are assigned to either 7^{th}- 8^{th} grade, 9^{th}-10^{th} grade, or 11^{th}-12^{th} grade students, or sets of elementary teachers to work with them in their problem solving groups or computer labs. Students in the eight-week program receive a stipend of $3,000 for the summer and those in the four-week program receive a stipend of $1,500 for the summer.

As one 2004 REU student put it: “While the main reason we were there was math, it was the community that we formed that made it possible (for me) to explore it (mathematics) at a level I had not seen before. The program not only provided me with a broader academic background but a new view that math is not merely an individual pursuit, but a pursuit to be shared and discussed with friends and others.”

It is likely that this is one VIGRE inspired activity that will last long after the grant is ended. Even though many universities have REU programs, this may be the only one that involves REU students in K-12 programs.

Research experiences for undergraduates are not new as a result of VIGRE programs; however, through VIGRE they have been increased and enhanced as is evident from the University of Chicago REU. Analogous research experiences for graduate students (REGs) are relatively new since 2000. After the start of its VIGRE project, the University of Illinois at Urbana – Champagne decided to use some of its VIGRE support to develop and maintain successful REGs.

Research Experiences for Graduate Students

The University of Illinois at Urbana –Champaign Mathematics Department Research Experiences for Graduate Students (REGs) are designed to engage beginning graduate students in the mathematical research agenda of the department and to thereby expand their preparation for participation in deeper research projects later on. Analogous to their counterpart REUs for undergraduates, REGs are summer research opportunities, designed for first and second year graduate students. REGs, developed during the five years of the UIUC VIGRE, have continued since VIGRE funds ended, through donations, even a large one from a very good poker player.

Some REGs work well immediately, others need more time; each has its own style. For example, Professor Scott Ahlgren worked with two students between their first and second years of graduate school in the summer of 2003 on two number theory projects. At the end of the summer, each student wrote up his own results as a paper, and submitted them for publication. According to these students, their projects successfully helped them gain a higher level of confidence in their ability to do research and a better understanding of the commitment necessary for scientific research. Professor John D'Angelo supervised a student in a REG who made no publishable progress during the REG, but when Professor D'Angelo got to know him better, he was able to direct him to another professor. This student finished his PhD in 2007 and now has a tenure track job. Professor Douglas West has run several larger scale REGs in combinatorics and graph theory. Four graduate students began work in 2006-7 on a number of open problems and are continuing their projects into the 2007-8 academic year.

REGs require a strong commitment of many faculty members in a department. As many as thirty-one graduate students have participated in a single summer UIUC REG program, some in multiple summers. Upwards of sixteen different faculty members participate in any one summer.

Most VIGRE Mathematical Sciences Departments integrate graduate students as early as their first year in research groups, or clusters, to provide them with opportunities to understand the process of mathematics research and to do research early in their graduate program. Examples of this type appear in Chapter 3.

2.2. Enhancing interaction up and down the ladder

Enhancing interactions across undergraduates, graduates, post-doctoral fellows, and faculty occurs most often while research and education are being integrated, and in other subtle ways. **In every instance VIGRE has made these interactions more visible and more sustainable.**

Research teams of undergraduate students, graduate students, post-docs, visitors, and faculty have matured through VIGRE projects and become very popular at many places, including the University of Maryland.

Research Interactive Teams at the University of Maryland

Research Interactive Teams (RITs) are particularly popular with mathematics students, post-docs, and faculty alike at the University of Maryland. With a large number of undergraduate majors (approximately 450) and a large number of graduate students (approximately 250) in the three departments of VIGRE (Mathematics, Applied Math, and Statistics), creating an environment where students and faculty regularly work together is important.

Each RIT team has a special research focus, and features projects appropriate to each member. Undergraduate students, and first and second year students are introduced to specialized areas of the mathematical sciences, such as cryptography, harmonic analysis, geometry, visualization, and more, in the RITs. Graduate students have found that they are able to more quickly move into research problems for their dissertation, thus shortening the time to degree. Post-docs through the RITs have a working community to continue their research, expand their horizons and mentor graduate students and undergraduate students. Mentoring and professional preparation are intrinsic to this structure, which emphasizes training in oral and written communication. In addition, frequent minicourses are given which enhance the RITs by providing short introductions to the research areas featured that year in an RIT.

By registering all the active research groups as RITs, and encouraging all students to participate in RITs, this University of Maryland VIGRE RIT structure also facilitates more effective and accountable advising and mentoring. There were twelve active RITs in the Fall of 2007 in diverse areas from *Statistics of High Dimension Data* to *Logic* to *Experiments in Geometry* to *Genome Sequencing.*

Juliana Belding spoke for many of her fellow graduate students when she said, "the RITs are great opportunities to communicate with other students and to practice talking about what we are doing. They really promoted a sense of working together with other students, and a sense of community within the Mathematics Department based on research."

A set of different examples, occurred at the University of Illinois at Urbana-Champaign during VIGRE and beyond. Two mathematics programs were formed to provide interpersonal support for students and strong mentoring at all levels – undergraduate, graduate, post-doc, and faculty. They are called peer groups, not in the sense of people of equivalent ages or place in their programs of study, but peer in the sense that they are bound together by common interests and objectives.

Across Level Peers (ALPs) and Research Among Peers (RAPs)

Across Level Peers (ALPS) groups consist of undergraduate students, beginning graduate students, and faculty focused on a particular topic of interest to students at varying levels. For example, the first ALP held in 2000 was organized around basic cryptography; another ALP that same year focused on information technology. Professor Bruce Berndt has led ALPS many years in q-series for graduate students and undergraduates. Several published papers are directly attributable to this ALP. More recently, topics have included robotics and geometry, engineering self assembly, and evolutionary game theory (joint with Political Science).

Research Among Peers – RAPs - groups are aligned with the research interests of both faculty and students. RAPs are focal groups for the VIGRE post doctoral associates. These are working groups that consist of faculty, post-docs and graduate students organized by post-docs. As many as 15 RAPs occur in any academic year. For example, in 2003-4, some of the RAPS held were in *Vector-valued Integration Connected to Economics, Metrics of Non-positive Curvature, Sub-Riemannian Geometry, Expander Graphs, Geometry and Algebra of Computer Vision, Supersymmetry, Sharp's Conjecture,* and *Computational Topology.* These groups focus on specific research topics and they become a way of understanding how one does research work and presents it in written and oral form. In some sense, RAPs are extensions of the old lecture seminars, thus the leap to the new form was not as difficult. The UIUC

Mathematics Department feels that RAPs were even more successful than the ALPs overall, especially since there were so many RAPS – over 75 in five years.

Many more examples of the integration of education and research appear in other sections of this chapter and in Chapter 3.

2.3 Broadening educational experiences

Broadening the educational experiences of students, to include workforce and early research opportunities, is a cornerstone of many VIGRE projects and a goal of all VIGRE projects. As was noted before, early research opportunities for undergraduates and graduate students are major activities of most VIGRE projects. Research clusters invariably include students at all levels at all of the VIGRE universities.

Some universities have developed undergraduate research labs, like those in the sciences, and linked them to a specific topic course. The Undergraduate Mathematics Research Labs at the University of Wisconsin Madison are linked to courses not necessarily found in the usual undergraduate curriculum.

Collaborative Undergraduate Research Labs

The University of Wisconsin (UWM) Collaborative Undergraduate Research Labs (CURLs) are linked to a topic course and taken for credit. CURLs are gatherings of undergraduates, graduate students, and UWM faculty engaged in exploring mathematical phenomena of all kinds. Whatever the specific CURL topic might be, the focus is always on genuine research questions that are appealing and accessible to undergraduates.

Undergraduate courses with collaborative research labs associated with them for the past four years and the professors involved include:

2003-4	Low Dimensional Combinatorics (Propp**)**
2004-5	Mathematical Biology (Kiselev, Mitchell, Milewski)
2005 Fall	Number Theory (Ellenberg)
2006 Spring	Mathematical Modeling of Biological Intelligence (Assadi)
2006-7	An Introduction to Cellular Automata and Interacting Systems (Griffeath)
2007-8	Mathematical Medical Imaging (Angenent)

As an example of how a CURL works, consider the Mathematical Biology CURL, where undergraduate participants meet for 4 hours a week, and are expected to invest roughly 6 additional hours per week outside of group meetings. Students, in the labs, are paid for their work on an hourly basis. Team-work and more particularly the sort of brainstorming that occurs when all group-members are present plays a pivotal role in the success of the research. Consequently, regular attendance is required. It is possible to take the Mathematical Biology course without participating in the CURL, but most students find them to be complementary and mutually reinforcing, as the course provides tools that will be helpful to students tackling research problems.

An example of a project from the 2006 Spring CURL is the following one by Laura Legault and Rebecca Yale. Their project is titled *Dancing through life: a geometric analysis of biological movement and dynamic gene expression,* and they provide the following abstract:

> Perception of biological motion has long been studied in psychology - humans can identify the activity and identity of moving objects from a few reference points. Similarly, from a crowd of reference points, one can determine which points describe an individual. With the assistance of the Dance Department, utilizing motion capture technology, they investigated the mathematical implications of this phenomenon,

discovered an analytic description of curves traced by human perambulation, and developed an algorithm (and computational implementation) to recognize patterns of motion in a group of points and classify the data into coherent subgroups. They worked on generalizing the underlying mathematics to model dynamics in complex systems, such as patterns of dynamics in gene expression, in order to discover constraints on the architecture of gene networks.

Each member of a CURL maintains his or her own CUR webpage with his/her calculations and a general "journal" of his/her research. In addition there is a group email aliases to further communications.

Few students learn of the range of opportunities for careers in the mathematical sciences while they are students. A number of the VIGRE sites have developed specific activities to fill this gap, including job seminars, resume development workshops, and internships.

Industrial Job Seminar

The University of Washington VIGRE organizes a Workshop on Working in Industry in late January each year. This workshop brings together students and post-docs from the three VIGRE departments – Mathematics, Applied Mathematics, and Statistics – with a panel of professionals from industry and national laboratories. The professionals describe their work experiences and how they got to their present point in their career.

As an example, in 2003, two professionals from Boeing, one a statistician, the other in numerical analysis, talked about the broad array of problems they work on at Boeing to support other parts of the company. Each spoke of the need for flexibility when working in industry. Steve Riley, of Microsoft, told the audience about the difference in type of problems one works on in academia versus industry. For example, developing encryption algorithms is important in academia, whereas correct and secure implementation of encryption algorithms is more important in industry. The same tools are needed for both tasks, however.

National laboratories are significant employers of graduates with advanced degrees in the mathematical sciences, yet few students know much about them. Van Henson, from Lawrence Livermore National Lab, described his career path, which included a job with the oil industry and theatre, prior to working at Lawrence Livermore Labs. He spoke of the excitement of implementing cutting edge algorithms on some of the world's fastest computers.

The collaboration with the Pacific Institute for the Mathematical Sciences (PIMS) further extends the breadth of experiences available to University of Washington (UW) students and post-docs. PIMS and UW hold a Post-doc Professional Development Day annually for all post-docs in the Pacific Northwest.

Numerous VIGRE sites have developed shared degree programs to provide cross-disciplinary opportunities for students. Because of VIGRE, the University of Colorado at Boulder Applied Math Department now has joint Masters Programs with Biology - an MS in Applied Math and a Masters in Computational and Developmental Biology. A student can also simultaneously work on a Masters in Applied Math and a Ph.D. in engineering or computational science.

Internships in government, industrial, and academic centers yield opportunities for students to experience the broad range of possible careers in the mathematical sciences and provide them with valuable research and development skills. CU-Boulder Applied Mathematics undergraduate students have had varied internships over the years of VIGRE and beyond. For example, four students have worked for Sun Microsystems, six for Milliman and Robertson, five for the Lab for Atmospheric & Space Physics, and others at NASA, NCAR, NOAA, NIST, Optek, Argonne, Sandia, and Lawrence Livermore Laboratories, Lehman Brothers, Microsoft, Goddard and Johnson Spaceflight Centers, and others.

The University of Wisconsin Madison internship program has been expanded with specific faculty designated as summer internship advisors. University of Wisconsin Madison graduate students in the middle of their graduate program have worked for Bank One, Motorola, Microsoft, Epic Systems, NCAR, the U.K. Oxford Center for Mathematical Biology, and Woods Hole, among others.

Additional specific examples of internships, both at the graduate level and the undergraduate level, can be found in Chapter 3.5.

Broadening the experiences of students has come to mean adding a number of other opportunities for students in addition to workforce and research opportunities. For example, the CAIN Project in Engineering and Professional Communications at Rice University was expanded to include students in the mathematical sciences through the Rice VIGRE. The CAIN project's goal is to prepare students as leaders, through the development of more effective communication skills, both written and oral. Communication instruction is integrated into the curriculum of the mathematical sciences courses as well as in separate courses. Project faculty work with faculty teaching the mathematics courses, offer workshops for faculty, graduate students, graders, and others, provide on-line resources, and make audio-video technology available. An extensive research project tracks the progress of the project and student performance.

The University of Michigan VIGRE provides a graduate student 4th year Communication Practicum to insure that doctoral students have the necessary communication skills to be successful in academia and industry. As Bob Lazarsfeld, Project Director for VIGRE stated, “this practicum is just one example of how VIGRE encouraged faculty to undertake innovative initiatives, both large and small of all sorts.” Ohio State University initiated a similar practicum in 2003 for the summer between a graduate student’s second and third, or third and fourth year. By 2005, this practicum at Ohio State University had merged with the seminar held during the year to prepare students for qualifying exams.

2.4 Motivating more students

Motivating more students, especially women and minorities, to study mathematics and statistics means starting early in the pipeline, and making the mathematical sciences attractive to those with other interests.

The North Carolina State University at Raleigh (NC-State) Statistics department provides an example of specific efforts directed at recruiting more students from underrepresented populations.

Recruitment

Recruiting women and underrepresented minorities takes a continuous, sincere, and sustained effort over many years at NC-State and elsewhere. Mentoring is an important part of NC-State’s Statistics Department investment in the recruitment and retention of students from groups that are underrepresented in the mathematical sciences. A key component of this effort is personalized mentoring to students.

For example, in recent years, Dr. Cavell Brownie, Dr. Jacqueline Hughes-Oliver and Dr. Kimberly Weems have served as mentors for African-American students, meeting them regularly to discuss research and educational issues. Dr.

Weems organizes monthly roundtable sessions for minority students. During these sessions, several successful role models discuss career opportunities and talk about how to overcome hurdles. Local statisticians and administrative professionals are invited to meet with the graduate students in the department to discuss their educational experiences and careers. These discussions are well attended, and the students benefit from interacting with accomplished minority professionals. Examples include Dr. Wanda Hill, Assistant Dean and Director of the Office of Multicultural Affairs, with advice on how to succeed in graduate school; Vikki Roy, a biostatistician at Lineberry Research, who discusses her career in clinical trials research, her day-to-day activities at Lineberry, and encourages the students to pursue the Ph.D. degree, and Dr. Steve Wisseh, Associate Director of Biostatistics in the Respiratory, Dermatology, and Infectious Diseases Therapeutic Area of the Department of Biostatistics & Statistical reports on his work at Novartis Pharmaceuticals Corporation in East Hanover, New Jersey.

Participation in MathFest, StatFest and SACNAS (The Society for the Advancement of Chicanos and Native Americans) conferences for many years has fostered the development of strong relationships with several women and minority serving institutions. In 2003, the Statistics Department organized a StatFest to attract underrepresented students to graduate studies in mathematical sciences. This was followed in 2006 by a Pipeline Issues Workshop for faculty of women's colleges and minority serving institutions, which focused on providing the information on careers in statistics. A second focus was to secure their feedback on how to improve the diverse workforce for future. The 2007 Conference on *Infinite Possibilities* celebrates the successes of women in the mathematical sciences from underrepresented groups.

The Statistics Department at NC-State has been highly successful in recruiting women and African-American tenure-track faculty, who serve as role models for faculty and students. Recruiting, coupled with excellent mentoring, is key to graduating a diverse pool of mathematical scientists, says the faculty in NC-State Statistics.

2.5 Summary

Vertical integration of education and research is clearly obvious at most VIGRE sites. Graduate students report interacting extensively with faculty and post doctoral fellows, and with the undergraduates in their departments. The Statistics undergraduates at North Carolina State, for example, report that the very accessible faculty and increased interactions with graduate students provides them with the ability and incentive to be successful completing a statistics degree. Time and time again, students at every level in VIGRE departments indicated that they felt that the presence of the VIGRE initiative motivated changes in the type and level of personal interactions across the educational spectrum are encouraging more men and women, majorities and minorities, to become students and to complete degrees in the mathematical sciences, especially when research is the focal point.

CHAPTER 3

Transformation - a Change in Culture

Large numbers of well prepared U. S. students have traditionally not beaten down the door of higher education in search of degrees in the mathematical sciences, either at the undergraduate or graduate level. Many people in and out of academia viewed majors in these fields as akin to "survival of the fittest" in a not-too-personal environment. VIGRE programs, to be successful, had to substantially change this perception and reality, by first and foremost transforming the way they do business in their mathematical sciences departments. Many have done just that in some cases dramatically so!

The reactants necessary for such change are becoming better understood as the VIGRE projects have evolved. They include leadership, strong mentoring, dynamic curriculum, high quality research and access to it, and internships.

3.1 Leadership

Everyone acknowledges that leaders are important for programs to be successful, and even more so when the program involves changing a long established culture. It is difficult to achieve purposeful, or sustainable change without strong leadership. There is clear evidence from the successful VIGRE sites that leaders emerged at these sites. In some cases these leaders are people who generated the proposal to secure funding; in other cases, they are people who assumed responsibilities along the way. When all is said and done, however, these leaders share authority in order to give people control of their own piece of the project, share responsibilities so they are not trying to do it all, and share credit for the successes along the way. The leaders of these successful VIGRE sites are not always who you think they might be. In all cases it has not been just one person, thus making long term sustainability truly possible. In some cases, Department Chairs or Heads assumed some of the responsibility of leadership for the VIGRE project in their department, but not always. There are projects where the leaders arose from the ranks of the faculty, and not just senior faculty, as is the case at the University of Arizona and the University of California at Davis. In other cases, a graduate student or postdoctoral fellow took on a heavy leadership role, as was the case at Princeton University where Steven Miller was a mathematics graduate student at the outset of the project; and at the University of Wisconsin Madison with Dan Knopf, a mathematics post-doc, who organized the undergraduate research lab on *minimal surfaces* early in the project. At the University of California at Davis, the post-docs organize and run the Research Focus Groups and the graduate students are in charge of outreach, as they are at the University of Washington and at the University of California at Berkeley in the Statistics Department.

At Brown University, mathematics undergraduates assumed leadership roles in VIGRE.

SUMS at Brown University

"VIGRE provided an infrastructure to empower students", says Tom Banchoff at Brown University. At Brown, undergraduates took it upon themselves to organize the Symposium for Undergraduates in the Mathematical Sciences (SUMS) and invite undergraduates from neighboring colleges and universities to participate. SUMS is designed to foster greater undergraduate interest and scholarship in mathematics by demonstrating the ubiquity of mathematics throughout the sciences and social sciences. It is also intended to introduce students to the full utilization of mathematics as a tool for the sciences.

The March 2007 SUMS Conference focused on areas of Pure Mathematics and Operations Research. It showcased research activities of undergraduates. For example, Adam Smith from Emmanuel College in Boston presented a paper entitled *What are the Determinants of Free Agents' Salaries in Major League Baseball* which indicated that the offensive statistics of a player are the primary determinant of a free agent's salary. David Hansen from Brown University presented his paper on *The Birch Swinnerton Conjecture* regarding groups of rational points on an elliptic curve and a special value of the Hasse-Weil L function attached to the curve. Key lectures are given by leading mathematicians and others are interspersed among the undergraduate lectures. In 2007, these were given by John Conway, David Dumas, Noam Elkies, Anna Nagurney, and Meinolf Sellman. The Chancellor of Brown hosts a banquet at the end of the day. SUMS 2008 focuses on mathematics and economics.

SUMS was initiated by undergraduates for undergraduates, and caused a transformation of the department as it integrated undergraduates into the working life of the mathematics department. The undergraduates maintain this momentum with an organization, begun under VIGRE, called DUG for Department Undergraduates.

Whereas Brown University provides an example of a mathematics department where leadership is assumed by the undergraduates, WOMP (Warm-up Program) at the University of Chicago is an example of where leadership is assumed by graduate students. In both cases, the students maintain ownership for their activities, which continue today.

WOMP at the University of Chicago

The Warm-Up Program (WOMP) at the University of Chicago was developed by, is organized by, and is run by graduate students for incoming graduate students in the Mathematics Department. It takes place during a two-week period of time just preceding the beginning of the school year. It is intended to:

- provide a bridge from undergraduate curricula to the very challenging first year program;
- introduce new students to older graduate students, who can provide mathematical and personal resources for them during their time as

graduate students; and

- introduce the new students to each other, the department, and the city of Chicago.

Started at the outset of VIGRE in 2000, it has become a permanent part of the Mathematics Department. It consists of ten to twelve two-hour sessions on mathematics. For example, in 2006, three days of lectures were concentrated on Algebra, and half days were devoted to Analysis I and II, Topology, Algebraic Topology, Manifolds, Category Theory, Manifolds I and II, and Homological Algebra.

Additional meetings, focused on the City of Chicago and the University, are held nearly daily and are coupled with social events in Hyde Park and around the city. For example, trips to the Field Museum of Natural History and the Art Institute are included, along with dinners at various restaurants. Many of the VIGRE projects have developed similar experiences for first year graduate students prior to the start of the first term, modeled after WOMP.

The University of California at Berkeley Statistics Department, which started their VIGRE grant later than some others, adopted the model produced by WOMP and uses it to orient first year graduate students to graduate work in statistics and the San Francisco area.

Leaders must be able to make decisions, thus they must have access to the decision makers of their university. In some cases, such as in the Statistics Department at North Carolina State University at Raleigh, one of the key decision makers, Dan Solomon, the Dean of the College of Physical and Mathematical Sciences, was a former member of the Statistics Department. This can make things easier or tougher, but at NC-State it helped make success attainable and sustainable. An obvious corollary to the need for decision making ability is that leaders must be credible both within the hierarchy of an institution and among their colleagues. Respect and rapport are key attributes of successful leaders at VIGRE sites. This is especially important for changes made during VIGRE years to be sustainable after NSF funding ceases.

Leadership is best when it is shared with others. Sharing leadership responsibilities accomplishes three major purposes – no one burns out too quickly, buy-in by larger numbers of faculty and students occurs, and the project does not collapse when one person leaves.   Collaborative leadership along with the leadership of the department helps to ensure continuation of activities after the money is gone.

3.2 Strong mentoring

Mentoring is one of the keys in successful VIGRE mathematical sciences departments. The concept of mentoring goes back at least to the days of Homer, where a mentor was described as a "wise and trusted counselor."(3) Today, mentoring is a personal, as well as, professional relationship between two people, designed to assist in the development of a successful professional. It is a relationship that develops over a period of time. Effective mentoring is characterized by mutual trust, understanding, and empathy. In these successful mathematical sciences departments, everyone is a mentor to someone else - undergraduate students mentor high school students, graduate students mentor undergraduate students, post-doctoral fellows mentor graduate students, and faculty mentor across and down the vertical ladder.

The North Carolina State University at Raleigh (NC State) Statistics Department has a well established mentoring program and culture begun under VIGRE, which includes undergraduates, graduate students, postdoctoral fellows, and faculty. This mentoring model, required of all faculty in Statistics at NC State, has been quickly transported to the college and the whole university.

NC State Mentoring Model

The NC State Statistics department seamlessly weaves mentoring into the culture of department life, by elevating it to the status of teaching, research and service in the sense that it is assigned, monitored via annual reports, and recognized and rewarded, in the same ways that good teaching, research and service are recognized and rewarded. The effect has been to establish mentoring as an essential component of every faculty member's responsibilities. As Dean Dan Solomon, Dean of the College of Physical and Mathematical Sciences says, “NC-State’s mentoring model creates a personal connection between undergraduates, graduate students, post-docs, and faculty”.

Post-doctoral fellows are assigned distinct teaching and research mentors just as students are assigned advisers. The teaching mentor is available to provide guidance on teaching and monitors one or more classroom sessions per semester, providing feedback as needed, as well as a report to the department head. Similarly, the research mentor is available for discussion about research topics, research funding and publication outlets.

Graduate students organized their own peer “buddy system,” where senior graduate students mentor new graduate students, especially during their first year. Particularly successful has been the summer qualifying exam preparation course, organized by and taught by graduate students who have passed the exam, for graduate students who anticipate taking the exam in the fall term. Faculty and post-docs also mentor graduate students in all academic and teaching areas. In turn, graduate students mentor undergraduate students.

In addition, certain curriculum changes that were made have, as an intended side effect, increased mentoring activities in the department, especially at the undergraduate level. For example, Professor Bill Hunt provides mentoring for all students in the undergraduate environmental statistics practicum. Typically one or more VIGRE graduate students or post-doctoral trainees are assigned to the practicum to provide mentoring to the undergraduate students. Vertical integration and mentoring occurs naturally within this group.

Mentoring has always been and will continue to be an integral component of the Ph.D. adviser-advisee relationship. However, even this long-established relationship evolved and co-advising has increased in the department. It often becomes apparent to the student that the advisers may have different opinions or approaches to a particular issue or problem, indicating that many issues are not clear cut, and students are encouraged to express their opinion more readily, facilitating communication between advisees and advisers.

Jason Brinkley, a graduate student at NC State in Statistics says:

> “VIGRE has been an essential part of my graduate studies – without it I would never have come this far. When I first started, VIGRE was there with programs involving older students to advise me on department expectations and time management. I got help from older graduate students in studying for my qualifying exams, and when I had questions about teaching and the classroom environment. These same mentors encouraged me to pursue an internship to gain real industry experience. They introduced me to the Summer Institute for training in BioStatistics which I eventually worked with and where I found my PhD thesis adviser. Now I mentor new graduate students.”

In speaking of the importance of mentoring, Sastry Pantula, Head of the Statistics Department at NC-State, says: “It takes an entire department to raise the next generation of problem solvers.”

Mentoring includes mentoring for teaching in addition to the more general mentoring among undergraduates, graduate students, post-docs, and faculty. For example, a structured intensive summer teaching workshop and academic year follow-up seminar were added to the available courses and seminars for *learning to teach* at the University of Georgia.

Georgia MathematicsTeaching Assistant Program

The University of Georgia Teaching Assistant Program in mathematics has three required seminars (workshops) for beginning Teaching Assistants (TAs) and a new program to help advanced graduate students and post-docs learn how to teach courses for prospective elementary and middle school teachers. Each of these seminars has become a permanent part of the University of Georgia (UGA) graduate curriculum. The *Introductory Teaching Seminar* is required of all graduate students in the fall of their first or second year at before they have responsibility for teaching their own section of precalculus or calculus. This seminar is led by a senior graduate student under the supervision of a faculty member. It meets once a week for two hours. Topics include testing and evaluation, academic honesty, time management, and how to deal with difficult or unmotivated students. Each student constructs a teaching philosophy statement and the students discuss the development of a teaching portfolio over the course of the semester. After performing a 15 to 20 minute practice lecture, students move into the classroom as guest lecturers for a day. These classes are videotaped so students can review the videos and reflect on their performance.

The second course, *Summer Teaching Workshop,* initiated in 2002 as part of the UGA VIGRE program, is required of all graduate students who will teach precalculus or calculus for the first time in the following academic year, and is also required of all advanced graduate students with research support in that summer term. Since many students received VIGRE research support in the summer, the Department took this opportunity to provide teaching mentoring as a complement to their research activities. The seminar lasts for two to three weeks, and meets ninety minutes a day. It has been led by senior faculty, by lecturers, and by post-docs. Several meetings involve discussions of case studies in teaching precalculus and calculus. The CBMS volume by S. Friedberg et al. is used for this purpose. Basic issues discussed include time management in and out of the classroom, grading homework and tests, balancing graduate student coursework and teaching preparation, academic dishonesty, and encouraging class participation. Experienced TAs share their experiences and each student.

The third course, *Precalculus/Calculus Teaching Seminar* is offered in the fall and spring, and is required of all students concurrently teaching precalculus or calculus for the first time. The Precalculus/Calculus Teaching Seminar was created in 2003 by Dr. David Galewski, who ran it until the summer of 2007. After his recent untimely death, several faculty have continued his work, indicating that it is a VIGRE activity that will be sustained long after the grant is over. The hallmark of this seminar is that it follows the associated course classes week by week, with specific and timely discussion of the structure of each class, homework assignment, and exam. TAs report that the seminar saves class preparation time. The seminar leaders observe each student's classes and review their exams. One student recently quipped that, "compared to the hands-on *Galewski seminar*, our other teaching seminars are more like *teaching therapy*." Leading this seminar is hard work for faculty, but they feel that the effort is richly repaid.

As Professor Clint Mc Crory, Project Director, adds about the linkages of teaching and research: "the supportive research atmosphere created by UGA VIGRE research groups over the last six years has fostered healthy discussions about teaching, providing an additional complement to the formal mentoring of teaching described above."

In some cases, mentoring is designed to include groups mentored in specific areas. For example, at Duke, graduate students are mentored in groups around ethics issues, including the requirement for third year graduate students to attend the Workshop in *Responsible Conduct in Research*. Post-docs at Duke are mentored in groups on grant writing and teaching, and undergraduates are mentored through their work on research projects.

3.3 Dynamic curriculum

Curriculum changes have been significant at both the undergraduate and graduate level. Curriculum has changed significantly at many places to include open ended inquiry early in undergraduate programs, to include courses designed to help graduate students learn to communicate better and to teach to broad audiences, and to include many more cross-disciplinary options.

A substantive curriculum that engages the students early and well in finding solutions to hard problems is a major part of many of the VIGRE projects. These changes necessitate even more faculty investment in the students' learning. As Professor Elias Stein at Princeton, winner of the 2002 Steele Prize and the National Medal of Science, said: "VIGRE was the impetus for faculty to do what they needed to do." When there were no self-contained books to guide students in their initial research endeavors, he, along with his colleague graduate student Rami Shakarchi, wrote them – four of them: *Real Analysis: Measure Theory, Integration, and Hilbert Spaces*; *Fourier Analysis : an Introduction*; *Complex Analysis*, and one not yet completed, as did two of his colleagues at Princeton, Steven Miller and Rami Takloo-Bighash. The Stein-Shakarchi books and the Miller-Takloo-Bighash book, *An Invitation to Modern Number Theory,* have been used successfully in similar programs at Brown University, Ohio State University, New York University, and other places, contributing to the transferability and sustainability of the progress of VIGRE sites. Graduate students at Brown, who read the Stein-Shakarchi books as undergraduates, praised them for their clarity of exposition and wealth of interesting topics and problems. More than 12,000 copies of the first three Stein-Shakarchi books have been sold, and more than 1500 copies of the Miller-Takloo-Bigdash Number Theory book have been sold for courses across the country (*Princeton University Press).*

Following the example of Princeton University in writing texts to engage undergraduates in discovery, The University of California at Davis VIGRE project has written a Linear Algebra text for that purpose. What makes all of these books different from traditional texts is the inclusion of accessible unsolved problems, laid out to be understandable to the reader with the requisite background material included in the book. Students can work on problems where there is no known solution – there is no answer in the back of the book.

As part of the emphasis on engaging students early in experimenting with solutions to hard problems, Princeton University developed Undergraduate Mathematics Labs.

Experimentation in Mathematics

The Undergraduate Mathematics Lab at **Princeton University** is designed to provide undergraduates with the opportunity to experiment numerically on a computer with a current topic in "pure mathematics". The developers of the Lab believe that this leads students to experience the subject in a concrete form, similar to the way things were first discovered, and to discovering some interesting phenomenon to be explored further. A Junior Research Seminar is taught alongside the Lab, where topics and projects are introduced to the students to help them explore mathematics first hand. The intent of the Seminar and Laboratory is to create research teams of undergraduates, graduate students, and faculty to investigate interesting unsolved problems theoretically and experimentally. As Professor Steven Miller says, the juniors leave with a much better understanding of what mathematicians study, how we attack problems, and whether or not they want to work in these fields.

How does it work? A Professor and a graduate student lecture twice a week for the first few weeks of the semester giving background on the general topic(s) and providing a list of unsolved conjectures that are amenable to experimental testing. For the first semester, the initial set of topics were the Hardy-Littlewood

Circle Method, Random Matrix Theory, Ramanujan Graphs, and Elliptic Curves. The undergraduates often have enough theory to understand the basic framework and proofs of simple cases. Building on this, they then numerically test the conjectures, dividing their time between research and writing and running the necessary programs. Class time is spent brainstorming and general discussions. They learn mathematical theory, coding, simulations, and optimization, and they get to see what is out there. The final component of the semester is a written report and a presentation to the class.

The Lab and the Seminar require a serious investment of faculty resources. An enormous amount of staff time is devoted to both the Lab and the Seminar – two professors and two graduate students (TAs) each term. As many as eight projects are often run simultaneously in one semester. However, the graduate students and faculty have a powerful calculating force for numerical investigations through these labs.

Building on the success of the Undergraduate Research Seminar and Laboratory, a Vertically Integrated Summer program in Computational Mathematics has run at the American Institute of Mathematics.

Often, working groups of faculty, post-docs, graduate students, and undergraduates have been developed into courses which students can take for credit at any level. For example, at Ohio State University, Professors Warren Sinnott and Vitaly Bergelson organized a working group entitled *Reading Classics*. This working group/course has been so popular that it has run every quarter since Winter 2003 when it started. Other Working Groups/courses occur, but this one in particular has been popular because of its accessibility by all levels of students and post-docs.

Reading Classics and More

The goal of the *Reading Classics Working Group* is to provide an intellectually stimulating seminar devoted to the study and discussion of the history of mathematics and classical mathematics texts. The intent is to study the way people did mathematics in the past and the impact their works have had on modern mathematics. The popularity of this Working Group has meant that each quarter, a different particular historical period or a particular mathematician is chosen for study.

Reading Classics meets each week for one and a half to two hours for the entire quarter. Each week, one or more of the participants gives a talk on the mathematics of the period or of the mathematician. The aim of these talks is to give insight into how mathematicians thought about their subject in the past, in order to provide even more insight into the process of research mathematics. Often, largely forgotten proofs of fundamental theorems appear. One example studied is Gauss's attractive first proof in his thesis of the Fundamental Theorem of Algebra, which uses properties of "algebraic curves". Participants write up short papers on their topic to share with the group.

Undergraduates, graduate students, and post-docs are able to deepen their knowledge, broaden their way of thinking, and experience communicating with diverse audiences through these Working Groups. The underlying mathematics of the classics is usually accessible to all of the participants – the difficulties are in understanding the perspectives, which have often shifted since the original work was written. A website catalogues the activities quarter to quarter of the Working Group.

These Working Groups, especially the Reading Classics Working Group have continued even though the VIGRE grant ended – a testament to sustainability. Spring quarter 2007 was devoted to Euler and Fall quarter 2007 was devoted to Newton. The prerequisites listed include an intellectual curiosity about the history of mathematics, the ability to use the library to research a topic, and the willingness to persevere in reading mathematics written from an unfamiliar perspective. Undergraduates are encouraged to have some background beyond calculus.

A related activity, loosely connected with this *Reading Classics* working group, is to produce readable modern English versions of various mathematical works: either papers that have not been translated into English, or older English works that would benefit from a modern treatment.

In an interdisciplinary context, another Working Group attractive to undergraduate and graduate students as a three-credit course has been and continues to be the *Mathematical Biology* Working Group. Some previous upper-level undergraduate mathematics experience is recommended, but no prior biological knowledge is assumed. The central goal of the working group is to demonstrate that mathematical modeling and analysis enable us to better understand the workings of biological systems. Related to this working group is the *Algebraic Statistics and Applications to Biology* Working Group in the Fall of 2007. Algebraic Statistics is concerned with problems that lie at the intersection of algebra, geometry, combinatorics, and statistics. Methods from algebra and geometry can be used to make statistical inferences; many statistical models for discrete random variables can be represented by classical algebraic varieties, e.g. secant varieties and toric varieties. The goal of this seminar is to understand this connection and its statistical consequences, such as in maximum likelihood estimation. Applications to computational biology, in particular to genome sequence analysis are studied. Further topics are determined by the interests of the participants.

Curriculum changes in VIGRE projects even included new courses designed to prepare graduate students to teach mathematics to a broad spectrum of student interests:

Teaching Future Teachers

Many faculty find themselves teaching courses for prospective teachers with very little preparation for teaching such courses. Therefore, the University of Georgia Mathematics department decided to provide *Mathematicians Educating Future Teachers* as an optional training for advanced graduate students and post-docs interested in teaching mathematics for elementary and middle school teachers. It is a two-semester program with an optional third semester. It is the brainchild of Dr. Sybilla Beckmann, who is a leader in the movement to involve mathematicians in teacher education.

In the first semester of the program, participants attend a mathematics content course for prospective elementary school teachers taught by an experienced professor. Participants write commentaries on their class observations, and these commentaries are discussed in the seminar. Also, participants observe several math classes at a local middle school or elementary school. In the second semester of the program, students teach a course for prospective elementary school teachers and meet at least once a week as a group with an experienced instructor. In the third semester, participants again teach a course for elementary teachers, and they act as mentors for other students teaching the course for the first time. The design of this program is intended to be time-efficient, so that participation is feasible for students and post-docs who are involved in research. Recent graduating PhD students who've been through Dr. Beckmann's training program have been highly sought after by mathematics departments who teach mathematics courses for prospective teachers.

Often, standard graduate curriculum consisting of Algebra, Analysis, and Topology are replaced by topic-oriented courses which emphasize the scope and unity of modern mathematics and its application, to better prepare graduate students for research today. For example, the cohesive core set of graduate courses at Columbia University now includes courses in Modern Geometry, Analysis and Probability, Complex Analysis, Groups and Representations, Commutative Algebra/Algebraic/Number Theory/Algebraic Geometry, and Algebraic Topology, with large overlaps and transitions from classical topics to state-of-the-art topics.

3.4 High quality research and access to it:

Research productivity is nurtured and increased at VIGRE sites. All of the VIGRE projects have built on their research strengths to involve those outside of the field as well as people at all levels of the educational spectrum. When students participate in research with faculty and other students within and across departments they experience the vibrancy of mathematics first hand – *they work on problems where the answer is not in the back of the book*. When faculty members work with students, they experience the unbounded excitement and creativity that those new to the enterprise engender.

VIGRE projects have built topic-specific research groups and they have called them by various names: research clusters, interactive teams, working groups, tetrahedra (faculty, postdocs, graduate students and undergraduate students are the vertices of a tetrahedron).

The Rice University Mathematics Department uses terminology from music, and calls their working groups fugues (spelled PFUGs).

Rice University PFUGs

PFUGs, pronounced fugues, at Rice University are a play on the term in music, where a fugue is a composition in which a subject is announced by one voice and then developed contrapuntally by each of two or three other voices. PFUGs are groups of undergraduate students, graduate students, faculty and post-docs devoted to the study of a specific problem area in mathematics and/or statistics and its applications. Between 10 and 20 people participate in each PFUG, typically 4-6 undergraduate students, 2-5 graduate students, 1-2 post-docs, and 1-3 faculty. Sometimes two PFUGs coordinate their work with each other. The participants typically come from more than one of the three departments in Rice's VIGRE (Mathematics, Statistics, Computational and Applied Mathematics), and in some cases they are joined by people from other departments. All PFUG undergraduate students must be declared majors in one of the three mathematical sciences departments.

Each PFUG lasts two or three years, sometimes longer, and as many as eight PFUGs occur in any one term. Essential elements of a PFUG include a Junior Seminar, a Research Seminar, appropriate supporting courses, and mentoring up and down the ladder by all participants. Faculty develop the intellectual theme and develop the curricular supports. Post-docs coordinate the Junior and Research Seminars, recruit participants, and mentor both undergraduate and graduate students. Graduate students mentor younger graduate students and undergraduates, and participate in all of the seminars, and preparatory coursework. The PFUGs are managed by the VIGRE Management Board.

In order to facilitate cross PFUG conversation RICE VIGRE-ON-LINE was formed, a dynamic on-line environment in which interdisciplinary groups working on parallel themes can achieve the full potential of vertical integration and the merging of education and research. It is designed to ensure that members at all levels will be able to contribute to work in many fields and appreciate the broad relevance of mathematical sciences. Several groups in the Rice VIGRE Program are working on similar themes (such as problems in gene networks, biochemical networks, and statistical genomics) and they learn from one another in order to “leapfrog” in their problem solving. However, interest across groups was at first impeded by lack of a central communication source and immediate opportunities to see what other groups are doing. Because participants in a group are from various levels and different majors and departments, a group as a whole cannot be assembled frequently to ask questions, demonstrate their techniques or problems, and engage one another—although interest in doing so is remarkably high. VIGRE participants are especially enthusiastic about their emerging projects and through Rice VIGRE-On-line they have been able to try out and play with one another’s equations and data sets, and help each other out.

Current and past PFUGs include Computational Algebraic Geometry, Developmental Biology, Decision Analysis, Physics of Strings, Computational Medical Imaging, Bioinformatics and Statistical Genetics, Computational Finance, Metabolic Engineering, Simulation-Driven Optimization, Geometric Calculus of Variations, Stochastic Processes in Molecular Biology, among others.

The Physics of Strings, for example poses the following questions:
How do strings vibrate and what forces slow them? How can one still a vibrating body as quickly as possible? Can one deduce material properties of a body from measurements of its vibrations? Participants are able to develop skills in:

- mathematical modeling - the development of partial differential equations that describe elastic and viscoelastic strings and twisted strings, with applications in acoustics and DNA;

- numerical analysis - high-precision spectral methods for solving differential equations, and tools from numerical linear algebra; functional analysis (analysis of the spectra and pseudospectra of string models);
- the interplay between mathematics and physical experiment - using equipment in the burgeoning string laboratory.

Moving students into research early provides a sense of ownership and discovery for students, two incredibly powerful motivators for staying with a difficult subject. Students, through early research opportunities, are able to master techniques, learn to think critically, acquire strategies for problem solving, and learn the importance of patience and perseverance. "Being a part of VIGRE at The Ohio State University and participating in the working groups helped me gain experience working with talented faculty, graduate students, and other undergraduates and gave me the opportunity to learn about my own strengths and weaknesses", says Julianne Wenger, a recent graduate of Ohio State University.

Some University VIGRE projects are particularly fortunate to have research centers close by that have contributed to their success. The University of California at Los Angeles VIGRE project has a very close relationship with the Institute for Pure & Applied Mathematics (IPAM), located across the street from the Mathematics Department. The overall mission of IPAM is to make connections between a broad spectrum of mathematicians and scientists, to launch new collaborations, to better inform mathematicians and scientists about interdisciplinary problems, and to broaden the range of applications in which mathematics is used. Every year IPAM offers two semester long scientific programs. These programs bring together senior and junior mathematicians, scientists, and engineers from the scientific disciplines related to the program. The overlapping interests of the UCLA VIGRE and IPAM make for very fruitful research collaborations among a much larger community than either could sustain by themselves. Mark Green, Director of IPAM says this of the interaction of UCLA VIGRE and IPAM:

> "UCLA has a wonderful climate for collaborative interdisciplinary training and research, and IPAM is a major part of that. It is this climate that makes it possible to train mathematicians at all levels in innovative and nontraditional ways, whether it is by attending a tutorial at IPAM, participating in an IPAM workshop or long program, mentoring or participating in a RIPS project or, across the way in the Mathematics Department, working as part of a VIGRE research cluster, attending the Distinguished Lecture Series, enrolling in one of the department's cross-disciplinary majors, or getting involved in one of the department's REU's. Another element in the success of these programs is the unusually strong working relationship between mathematicians at UCLA doing pure mathematics and those pursuing applications and those doing both, enabling the UCLA Math Dept to foster interdisciplinary interactions while continuing to run a first-rate program in pure mathematics. Conversely, the activities at IPAM and VIGRE have considerably strengthened both the climate and relationship just mentioned, with the result that the rest of the campus has a highly positive view of our department. There is thus a virtuous circle where innovation leads to an even more receptive climate for further innovation. UCLA is not alone in this. We mathematicians are living through a kind of Golden Age for mathematics, both in the extraordinary advances within the field and in its deepening impact in a rapidly expanding number of other domains, and I see this as manifested in the role that mathematics departments are playing on more and more campuses across the country."

3.5 Availability of internships

Broadening the educational experiences of graduate students, as well as undergraduate students, motivates students in the mathematical sciences. For example, industry and government agencies, such as the Motorola, NCAR, and the EPA, provide problems and internships for students in vertically integrated teams of undergraduate students, graduate students, and faculty at the North Carolina State at Raleigh and at the University of Wisconsin. In addition, NC-State Statistics graduate students provide free statistical consulting for the campus community in order to gain further experience.

UCLA's summer internships for mathematics graduate students are varied and very popular:

Graduate Internships

UCLA has internships as a major component of its VIGRE efforts to broaden the experiences of its graduate students. Faculty from other departments on campus and at other places are eager to work with mathematics graduate students. The students are enthusiastic and successful. For example, Joseph Busch participated in 2005 and 2006 in summer internships at UCLA. Working with Professor Marcus Kracht from the UCLA Linguistics Department he spent his first summer on a project titled "Lambda-calculi and type theory". Lambda calculus is a formal language with historical roots in the early development of recursion theory in the 1930's. Today, it has many applications in computer science and linguistics. Work from that summer has appeared in *Fundamenta Informaticae.* Continuing with applications to computer science the next summer, he worked with Professor Elias Koutsoupias in the Department of Informatics and Telecommunications at the University of Athens, on a project entitled "Lower bounds for higher types". Modern programming languages encourage the use of higher-type functions with the hope of improving efficiency. However, they discovered that for some measures of complexity, there is a limit to the gains in efficiency obtainable.

Yifei Lou, provides a second example of a student internship opportunity at UCLA. She worked with Stefano Soatto in the UCLA Vision Lab in both 2006 and 2007. They study a model of shape, motion, and appearance of a scene that captures occlusions, deformations, and changes in its radiance. The model is illustrated on synthetic image sequences to show its performance in special cases. She says, "I found it good to absorb the knowledge and thoughts from a non-math area to broaden my views on the applications of mathematics and to stimulate my research in applied mathematics." Professor Soatto became her PhD thesis co-advisor as a result of these summer experiences.

Colin Hinde, a mathematics graduate student, worked with Electrical Engineering Professor Yablanovich, and Applied Mathematics Professor Caflisch designing a plasmon producing channel of "optimal geometry" defined in terms of power loss, in 2005. A plasmon is a waveform produced by the excitation of an electron gas, with potential for extremely low wavelengths with less energy than electromagnetic waves of the same length... "Since I am not an Applied Mathematics major, I learned a great deal of physics and mathematics and provided a more diverse background for my graduate work", says Colin.

Internships in statistics and applied mathematics are often easier to accomplish than in pure mathematics. Every one of the more than one hundred statistics undergraduates at NC- State in Raleigh, North Carolina is required to participate in a research/consulting internship. These internships provide the students with examples of the types of careers statisticians working for the government and industry can expect will be available to them when they graduate with a degree in Statistics. As one student put it, "while the dive right in approach might be intimidating to some, I found it to be invigorating and most stimulating." Another student said that even though it was an industry internship, it encouraged him to go to graduate school.

Undergraduate Internships at NC-State Raleigh

The Environmental Statistics Practicum at NC State, Statistics 495, is the major vehicle used for linking statistics undergraduates and clients. The objective of this practicum is to provide an opportunity for students to work on a consulting project with a researcher/client using real environmental data. As Dr. Ellis Cowling, NC-State University Distinguished Professor at Large said of the program in an Academic Review Board report:

"Wider implementation of the Environmental Statistics Practicum

> idea could lead to a renaissance in what can be learned from environmental data that has already been collected and will be collected in the future. It is a win-win-win opportunity for all parties involved: undergraduate and graduate students, university faculty, and various federal and state government agencies and private-sector clients."

Examples of clients of the program include the U.S. Department of State, Environment Canada, USEPA's Offices of Research and Development, Air Quality Planning and Standards, and Environmental Information, RTP Environmental Inc, and the Midatlantic Regional Air Management Association.

Fawn Hornsby and Wilma "Billie" Jackson, undergraduates, under the mentorship of Professor William Hunt of the Department of Statistics at NC-State, worked with Charles Pietarinen of the NJ EPA to collect and analyze data on air pollution due to mercury acquired in Elizabeth and New Brunswick, New Jersey as part of their internship. They looked at mercury, a toxic pollutant that causes adverse health effects in humans, and its release into the atmosphere from industrial sources and subsequent entering into the water supply where it accumulates in fish, which humans consume. Elizabeth, NJ and New Brunswick, NJ are both industrial cities with mercury monitoring sites which use Tekran Continuous Mercury Analyzer technology. They were able to show that skewed right diurnal patterns existed for each of the three forms of mercury pollution – elemental, particulate, and reactive gas mercury. Fawn and Wilma were able to help New Jersey set a research agenda to study the effects and generation of mercury pollution for years to come. They were able to see the value of working as a statistician to help address environmental health issues.

Louise Camalier, Brendan Yoshoimoto, and Bryan Stines, also undergraduates at NC State, working with Professor Hunt, Candy Garret, and Erik Gribben of the Texas Commission on Environmental Quality in Austin, and Van Shrieves of the U. S. EPA Region 4 in Atlanta, developed a statistical method for air quality data to corroborate volatile organic compounds and nitrogen oxide emission inventories in the air from refineries and chemical plants in Houston, Texas, and Atlanta, Georgia. The Environmental Protection Agency was interested in knowing if ambient data can be used to corroborate emission inventories, and how one deals with outliers due to industrial upsets in the data. Louise, Brendan, and Bryan were able to definitively answer both questions through the statistical methods they developed to address the problem.

The use of internships, as well as early opportunities for research, has increased the retention of mathematics undergraduate majors, broadened the perspective of graduate students and has brought both undergraduates and graduate students, into direct contact with real world problems and students and faculty from other disciplines. Meeting the workforce challenges of this century requires mathematical scientists at all levels for industry, academia, and the government.

3.6 Summary

These examples of VIGRE departmental and university success stories provide insight into some of the main components of what it takes to create positive change in culture in mathematical sciences departments. This change is to a culture that simultaneously supports both research and education, and better integrates undergraduate students, graduate students, post-docs, and faculty towards the goal of high quality research and education. Leadership, mentoring, curricular change, access to high quality research, and internships are key ingredients. Undoubtedly there are more ingredients that contribute to overall success, but these turned out to be common to most VIGRE projects and evidence suggests that without them, success is not nearly as possible.

CHAPTER 4

Connections - the Reach is *Long*

The impact of these VIGRE mathematical sciences departments has been felt beyond the individual departments funded. These departments have reached out to other departments, to colleges and universities, to K-12 students and teachers, and to the whole greater community.

4.1 Networks

Crossing disciplines and crossing colleges and universities extends the reach of VIGRE projects and research opportunities in the mathematical sciences well beyond a single department. Duke University, the University of North Carolina at Chapel Hill, working with North Carolina State at Raleigh, as they do in VIGRE, are a stronger statistics research entity than any one of them can be by themselves. The University of Washington works closely with institutions throughout the region, including high schools, to grab talented young people for the mathematical sciences and foster their development from high school through graduate school and into the work place. Similarly, the University of Arizona has the UA VIGRE Southwestern Network, which includes 19 colleges and universities in six states, as a major component of its work.

Transferability - Mathematical Sciences Networks

The University of Arizona VIGRE Southwestern Network is designed to foster a broad exchange of ideas and people throughout a six state region in the southwestern United States. Ultimately, the Network aims to increase the number of talented students from diverse backgrounds who pursue advanced degrees in the mathematical sciences, while at the same time providing for a lively interchange of ideas on research, education, and mentoring, among students and faculty. Colleges and universities in Arizona, California, Colorado, Nevada, New Mexico, and Texas participate in the Southwestern Network.

The network functions on an informal basis, with people and institutions participating to the extent of their interest. Meetings are held twice a year, often in conjunction with other events such as a regional AMS or MAA meeting, or other research conferences. Network faculty and graduate students are given first choice to participate in the Arizona Summer Program along with undergraduates from schools in the Network. It is expected that through experiences with the Arizona Summer Program, Network colleges and universities, develop research experiences for undergraduate students at their own home institutions. Modules developed through collaborations in the VIGRE project are easily exportable to all of the Network colleges and universities.

Better preparation of students for careers in the mathematical sciences from all of the institutions in the network is fostered through faculty professional development programs and meetings of the Network with the goal of increasing the depth, breadth, and quality of education in the mathematical sciences at all of the colleges and universities in the Network. At the same time graduate students and post-docs become better acquainted with the range of colleges and universities in the region and the issues these schools face on a daily basis, as they begin to look for jobs.

A relatively new VIGRE program at the University of Iowa formed a network of neighboring colleges and universities in the Midwest, called the *Heartland Mathematics Partnership,* to increase the number of highly talented students entering the mathematical sciences workforce. This partnership includes The University of Iowa, Bradley University, Central College, Clarke College, Coe College, Cornell College, Grinnell College, Loras College, Luther College, Truman State University, University of Wisconsin Eau Claire, University of Wisconsin La Crosse, and Wartburg College.

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4.2 Reaching students in K-12 schools

Filling the pipeline of mathematical scientists for the future starts early, at least as early as high school. Motivating students early to study mathematics and statistics, and keeping them motivated is a shared responsibility between schools and the university. Though not a specific requirement of VIGRE, many VIGRE programs realized early that one way to motivate more students to study mathematics and to increase the number of U. S. students engaged in graduate work in the mathematical sciences is to engage the university mathematical sciences departments with high schools in the area. Faculty, graduate students, and undergraduate students work with the K-12 community in such areas as Math Fairs (University of Washington), Math Circles (University of Utah and California at Davis), WRAP (Rice University) and various high school summer programs such as the ones described at the University of Chicago for students and teachers.

VIGRE K-12 at the University of Chicago

Postdocs, graduate students, and undergraduate students on a volunteer basis during the academic year, and undergraduate students as part of the summer REU program play essential roles in four K-12 education programs run by the mathematics department at Chicago. In the process, they are introduced to a wide variety of eye-opening teaching experiences at varying levels.

YOUNG SCHOLARS PROGRAM. This program brings large numbers of students in grades 7 through 12 to the University of Chicago for a summer mathematics enrichment program. There are three tracks, each with two rotating courses, at least one in mathematics and the other often in a related area of application in the physical sciences. Each course is accompanied by a computer laboratory. There is an accompanying Saturday morning academic year program.

SESAME: SEMINARS FOR ELEMENTARY SPECIALISTS AND MATHEMATICS EDUCATORS – This is a three-year, 270-hour, program for elementary school teachers in the Chicago Public Schools. It offers 90 hours of classroom instruction each year, 60 in the summer and 30 during the academic year. Participating teachers earn appropriate formal educational credit. The summer program is mainly focused on mathematics directly relevant to classroom teaching; the academic year program focuses more on mathematical enrichment.

SUMMER SEMINAR IN CALCULUS. Around 20 high school teachers and 20 high school students come to the University to participate in this program, which focuses on the calculus and its applications. One of the ideas behind the mix of teachers and students is to convince the teachers that calculus can be taught to high school students.

SEMINAR PROGRAM FOR HIGH SCHOOL TEACHERS. Around 20 high school teachers attend this summer program, which offers seminars in algebra and geometry, complementing the SSC program in calculus. The material taught is considerably more ambitious and demanding than the standard curriculum that these people generally teach.

These K-12 Mathematics programs at the University of Chicago are by no means new – the Young Scholars program dates back to the 1960's – however VIGRE changed the degree and type of faculty and student involvement in the programs. These programs are stronger as a result of VIGRE and have even more potential for sustainability long term.

4.3 Community

In addition, some universities have found some very creative ways to engage more students and faculty from other departments in problem solving.

One interesting example of the latter is the University of Washington's *Problem of the Week,* also called *Challenge of the Week.*

Challenge of the Week

The Challenge of the Week was started in Fall 2006, and organized by Professor Bernard Deconinck, faculty member in the University of Washington (UW) Applied Mathematics Department. Professor Deconinck, working with a group of VIGRE fellows from all three departments (Statistics, Applied Mathematics, and Mathematics), pose a mathematical problem weekly to the whole University of Washington at Seattle community, and beyond, through the website: http://www.ms.washington.edu/challenge. Problems are meant to be understandable to almost everyone on campus with their solution not requiring more than the ingenious use of calculus or lower level mathematics. The goal is to increase the visibility of the mathematical sciences on campus, and to attract students to major in mathematics that might otherwise have not considered the option. Professor Deconinck used this vehicle successfully at Colorado State University, and felt that UW could be equally successful using it as part of its VIGRE program. He was right.

The average page load/day is 102, with 54 unique visitors each day. The number of average correct solutions for each problem is 15.6 (with mostly undergraduates and some graduate students and faculty from inside and outside the university). Peak days (when a new problem appears) have seen as many as 300 visitors to the web site. Approximately 29% of the participants have been from outside the university, and one third has been from outside the mathematical sciences.

The winner each week is drawn from the set of correct solutions with preference given to undergraduates. Winners get a five dollar gift certificate from

Mix Ice Cream, sponsor of these challenges. In the long run, the Challenge of the Week is an area-wide activity, through interactions with other local institutions. Over 85 different problem solvers provided correct solutions to problems posed weekly in 2006-2007. In 2007-08 advertisements will appear more broadly in local newspapers - right beside the Cross-word and Sudoku puzzle! Professor Deconnick says, "such an activity is an easily sustainable cross-departmental endeavor, however time consuming for the one who manages the process." Undergraduates, graduate students, and faculty can take turns at managing the *Challenge of the Week*.

An example of a challenge of the week problem is the first one which appeared in the autumn of 2005. It received 61 correct solutions:

> In a cryptarithm, digits of numbers are replaced by letters; a given letter consistently represents the same digit and different letters represent different digits. Solve the following cryptarithm:
>
> ONE and NINE are odd perfect squares and FOUR is an even perfect square.

A second quite different example is the following problem posed in Spring 2006:

> Suppose **A** and **B** are square matrices of the same size and **ABAB** = 0. Does this imply that **BABA** = 0? Prove it if it is true or provide a counterexample if it is false.

In a similar vein, *Math Circles* at the Universities of California at Davis and the University of Utah, engage ninth through twelfth graders in exploring new mathematical topics and challenging problems. *Math Circles* originated in Hungary more than a century ago and have helped produce some of the world's greatest mathematical minds. Students in *Math Circles* work with professors and graduate students weekly, and apply their new knowledge to solving interesting problems. Contests are held to let students test their grasp of the topics and win prizes. One high school student participating in the *Math Circle* at Davis remarked that “I like *Math* Circle and I don’t even like math.” *Math Circles* provide a sustainable setting for high school students to discuss mathematics.

Most VIGRE sites have outreach activities that may last a day, a weekend, a quarter, or whole year, and are open to the public. For example, Math Fest at the University of California at Davis hosted Dr. Tony De Rose of PIXAR Studios for an evening of Math in the Movies, followed by games and puzzles, during the summer of 2007. Columbia University reaches out to the large pool of talented students in the New York City area through its Saturday Science Honors Program and to diverse students in the city through the Double Discovery Center.

4.4 Summary

The reach and influence of VIGRE extends well beyond the university departments who received NSF funding, all the way to grade schools, high schools, community colleges, small colleges, and large universities. VIGRE mathematical sciences departments will have had an impact on the academic and industrial workplace well into the future.

CHAPTER 5

A Focus on Education Coupled with Research Up and Down Vertical Levels Does Make a Difference in Numbers

The answer to the question of whether emphasis on education for all and by all makes a difference is a resounding yes. More importantly, it has translated into increased numbers of U. S. students entering undergraduate and graduate mathematical sciences programs in a number of places in the United States. Both the quantity of students and the quality of students in many of these programs has increased over the last few years. More women and more minority students are entering the mathematical sciences, recognizing that they are talented, welcomed, and supported.

5.1 Undergraduates

Examples exist where dramatic increases in numbers of U. S. citizens are entering and completing degree programs in the mathematical sciences. Perhaps the most striking example is at the University of Chicago. There, Mathematics jumped in terms of numbers of majors so much that it now ties with English Language and Literature as the fourth most popular major. In the spring of 2001 (the first year of VIGRE) there were 141 mathematics majors; in the spring of 2006 there were 227. The number of women undergraduate mathematics majors more than doubled during this period. As Professor Peter May at the University of Chicago says, “plentiful exposure to opportunities to learn and to do mathematics and also plentiful opportunities to teach mathematics will lead to a plentiful supply of bright young people entering our field (mathematics).” He put this optimism to the test and he was rewarded.

The graph in Figure 2 shows increases for six universities where data indicates a doubling or near doubling of undergraduate mathematical sciences majors during the years the VIGRE program was in place. Other VIGRE sites have seen increases in undergraduate enrollments, but not nearly as dramatic.

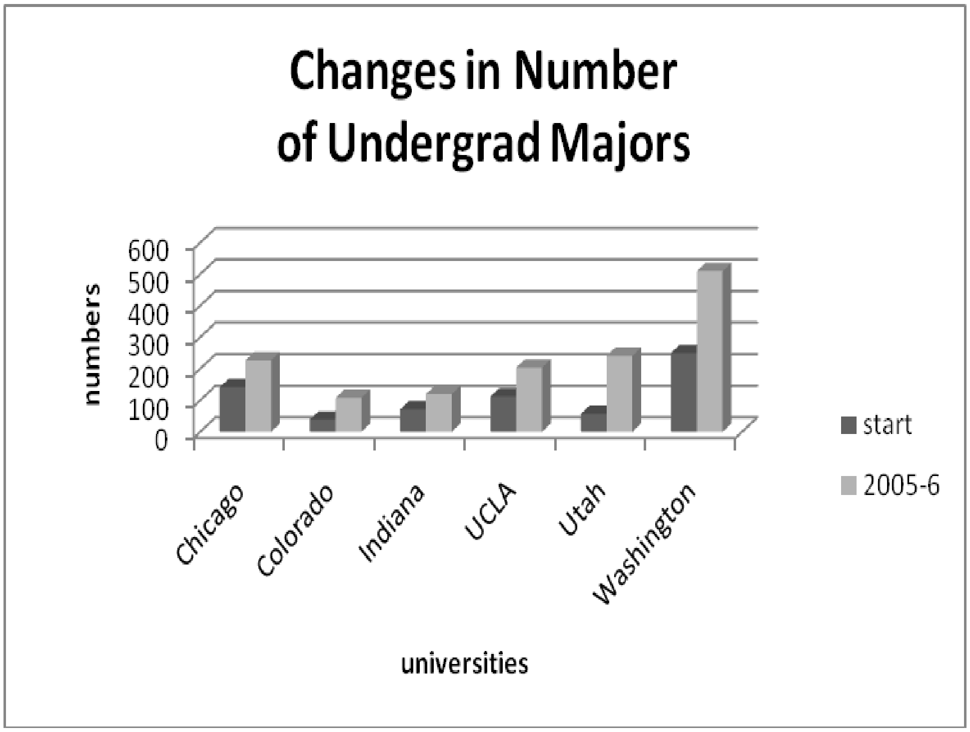

Figure 2: Changes in the number of undergraduate mathematical sciences majors

Source: Data provided by VIGRE universities

Another quite different example is provided by the University of Colorado Applied Mathematics Department. The number of Undergraduate majors increased from 39 in 1999 to 108 by 2004. The department attributes this change in undergraduate majors to the sense of community engendered by the VIGRE opportunities for undergraduate students to participate in research collaboratively with graduate students and faculty early in their undergraduate program.

At the University of Washington, enrollments doubled, going from 250 undergraduate mathematical science majors at the start of VIGRE in 1999 to 510 mathematical sciences majors, five years later. According to Doug Lind, former Project Director of VIGRE**:**

"We believe a far richer set of opportunities, including numerous cross-disciplinary activities, contributed to the jump in enrollments of U. S. students and the doubling of the number of mathematical sciences majors at the University of Washington".

5.2 Graduate students

Mathematical Sciences graduate enrollments of U.S. students have also increased sharply in universities during the VIGRE time-span. Figure 3 shows percentage increases in U. S. graduate students in mathematical sciences departments for four universities with VIGRE grants. The University of California at Los Angeles (UCLA) indicates a 75% increase and the University of Utah indicates a 66% increase in the number of U.S. mathematical sciences graduate students since 2000. The University of California at Davis and Indiana University show a doubling of the number of U. S. graduate students, from 45 to 90, and from 35 to 79, respectively, with a corresponding increase in women graduate students. The number of women awarded Ph Ds at the University of Chicago went from zero at the beginning of VIGRE to seven in 2007.

Figure 3, which follows, shows the percentage increase of US graduate students since the start of VIGRE at a sample of four universities. All VIGRE supported departments showed some increase in the number of US graduate students, but these four had the most dramatic increases.

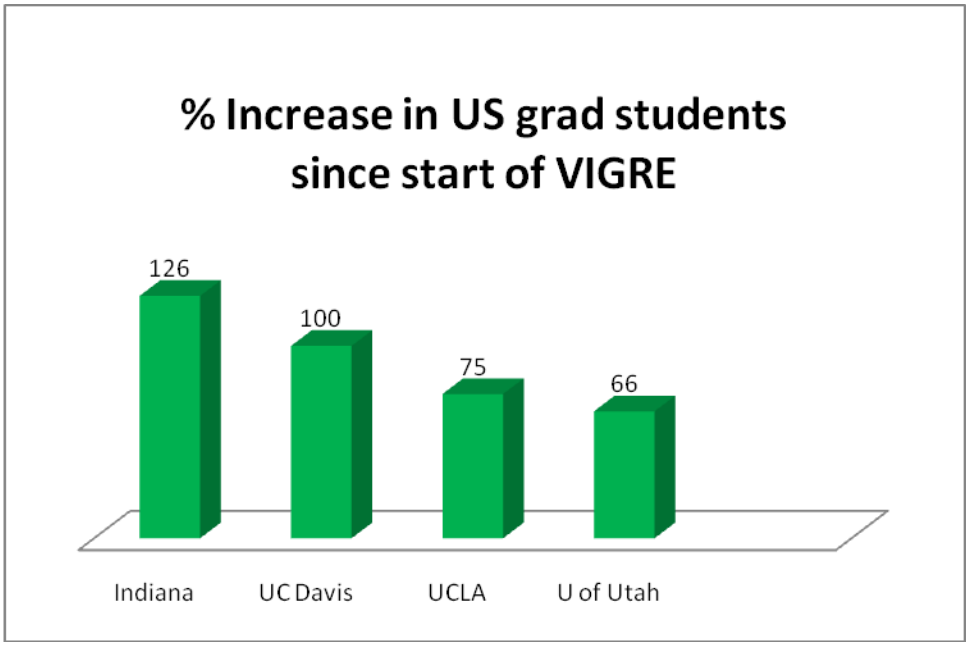

Figure 3: The percentage increase in US graduate students since the start of VIGRE

Source: Data from the universities

5.3 Postdocs

VIGRE sites included support for between one and five post-docs in their budget, usually for a period of two-three years for each individual. Even though about half of the universities have named post-docs in addition to the VIGRE post-docs the VIGRE program dramatically expanded the total number of post-docs in the mathematical sciences nationwide. NSF required that VIGRE post-docs go to U.S. citizens. The named post-docs could then be used for non-U.S. citizens.

Post-docs are a big expense, but faculty and the post-docs believe they offer a positive return on the investment, both for the post-docs and the colleges and universities where they will eventually teach, or industry where they might find a job. VIGRE post-docs have been quite successful in securing jobs at the completion of the post-doc. Lewis Bowen, a post-doc at UC Davis from 2002-2004, now Assistant Professor at the University of Hawaii, indicates that his post-doc helped his career for three main reasons: “(1) I was frequently exposed to the latest research ideas at conferences in and around UC Davis, especially at MSRI; (2) the teaching load was relatively light which helped me to develop my teaching style while not taking too much time from research; and (3) I met many researchers near UC Davis, some of whom I have had serious collaborations with and who wrote letters of recommendation for me.”

Perhaps, one of the most important outcomes from holding a VIGRE post-doc is the ability to work in all aspects of the VIGRE program, running research experiences for undergraduates, mentoring graduate students, participating in research groups, and doing outreach of many forms. Post-docs gain a greater insight into what is possible at different types of colleges and universities. For example, Alex Woo, another post-doc at UC Davis, wants to teach at a small liberal arts institution and run a research focus group for undergraduates there like the ones he worked with at Davis. He also remarked on how much he was treated with the same level of respect as a faculty member while a post-doc at UC Davis – this gives him more confidence as he moves to a full time teaching position. The lessons he learned through VIGRE at UC Davis he plans to apply in his new position.

Teaching, in addition to research, is a requirement of all VIGRE post-docs in order to give potential faculty greater exposure to the full set of undergraduate and graduate courses, and to manage an undergraduate course independently. VIGRE at Texas A&M helped foster excitement about teaching as well as enhance their interactions with faculty. One of the comments from a mathematics post-doc at Texas A&M tells this story: “Each of the students in the VIGRE seminar I ran showed noticeable improvement over the course of the semester, and the feedback I received from the students was quite encouraging. This was my first professional experience in the role of adviser and I found it rewarding .”

Aaron Jaggard, currently a new faculty member at Rutgers University, formerly a VIGRE mathematics post-doc at Tulane University, says this of his experience as a post-doc at Tulane:

> "I really enjoyed and benefited from the range of responsibilities I had and the independence that I was granted as I fulfilled them. Over the four years, I taught only one course twice; at times I disliked preparing something new each term, but overall, I appreciated the variety. I was able to develop a broad teaching portfolio within the safety and comfort of a one-course per semester load. While my senior colleagues were always willing to help me with questions on how to approach a course, I had the freedom to put my own stamp on course at both the graduate and undergraduate levels, including designing a new undergraduate course which has now become a regular offering. "

Post-docs come from a wide variety of graduate school backgrounds. For example, some graduate schools provide extensive experience developing proposals, writing and giving talks, working with a broad range of disciplines, and mentoring undergraduates and graduate students early in their program, while others do not. The VIGRE post-docs at most VIGRE sites get to gain all of this experience as a post-doc and more. Aaron Jaggard continued his description of his experiences at Tulane as follows:

> "I ran an REU for five students one summer and have continued to be involved in the subsequent work of two of them; in addition supervised senior projects. This work reinforced my view – developed during my own time as a student – that such interactions significantly enhance the education of undergraduates – it also helped me think how I will advise students as my career progresses.
>
> At Tulane I was required to write a proposal; by the end of my post-doc, I had written six, five as sole PI, and two with people at other institutions. Beyond providing funding and experience with the proposal process, proposal writing has even sharpened the direction of my research. Now I view proposal writing as a part of my responsibilities.

A number of VIGRE sites have secured a commitment to continued funding of some of the post-docs, once VIGRE funding goes away, but this is not the norm. The challenge for most sites will be continuing to recruit U.S. post-docs without a dedicated funding source.

5.4 Summary

Though limited to a few universities, the data provided in this chapter suggests that the mission of the VIGRE program to increase the number of US students completing work in the mathematical sciences and ready to enter the workforce has increased. This author believes that these numbers would be even stronger, had data collection been better at all of the VIGRE sites from the outset. The VIGRE university mathematical sciences departments strongly believe that the quality and the diversity of the students in their programs at all levels have increased.

CHAPTER 6
CONCLUSION

It is possible to draw a number of conclusions from this set of success stories. The most important conclusion has two parts. First it is possible to integrate research and education so that both areas gain substantially. Secondly, undergraduate and graduate students have an enormous amount to offer to any mathematical sciences department; they only need to be provided with opportunities and challenged to develop activities and request resources to meet their needs.

The fact that some things don't cost very much, but they make a huge difference in the long run, stands out among VIGRE sites. The Challenge of the Week at the University of Washington may be time intensive, but costs nothing. Using summer REU students (Research Experience Students) to counsel and work with high school students is a win-win for both sets of people and costs less than if other people were hired to do the task. Also, this relationship puts front and center the inextricable link between research and education.

Empowerment of groups at every level is critical. When undergraduate students and/or graduate students are empowered to both propose activities and run them, everyone gains. The Department has others to share the work load, and everyone participates and takes ownership. For example, many of the VIGRE sites have late summer programs for first year graduate students, run by graduate students, to acclimate them to the graduate program. This carries over to programs to help students prepare for qualifying exams, developed and run by graduate students who have already passed the exams. A surprise to many has been how capable undergraduate students are in recognizing their own needs and what will help them meet these needs. Undergraduates have organized conferences, developed programs to assist high school students, participated in research groups, and assumed leadership roles on curriculum committees.

Early undergraduate research experiences make a big difference to them and to the research of the department. Faculty tell of students, as early as their freshman year in some instances, working on problems that they as faculty members had attempted to solve, and having the students come up with solutions. Anne Dougherty, Associate Department Chair of the University of Colorado at Boulder (CU-Boulder) Applied Mathematics Department, tells her view of the value of these early research experiences:

> "Experiences in the department over the past decade strongly support the proposition that undergraduate research projects early in the careers of students can be life changing. Students are recruited as freshmen through honors seminars. Once students become sophomores and juniors, they are invited to join small research seminars led by faculty and advanced graduate students. After the seminars the students may continue their own research projects alone or in small groups. Students learn that they can make unique and original contributions to the mathematical sciences, and many are motivated to continue their education by taking more challenging courses and continuing on to graduate school. As a consequence, CU-Boulder applied math alumni can be found in some of the most prestigious graduate programs in the country."

6.1 Sustainability

A key question to ask is whether or not the successful activities of these mathematical sciences departments are sustainable once the VIGRE program funding ceases. Conversations with Department Chairpersons and Deans indicated that the answer is a clear yes to those activities that are not too expensive, such as an undergraduate conferences, Boot-camps for graduate students prior to the beginning of graduate work, the Challenge of the Week, the Arizona Network, just to name a few.

In some cases, where the activity is more extensive and expensive, such as Research Experiences for Undergraduates, sites are able to acquire grant funding from other grant programs, including those at other agencies. For example, undergraduate research was funded in the Applied Mathematics Department at CU-Boulder during VIGRE. The VIGRE grant ended in the spring of 2006, and now the department has a funded initiative which builds on the successes of VIGRE, called *MCTP: Colorado Advantage*. This new initiative focuses on helping students evolve from "good students" to independent learners and researchers, just as was done through VIGRE.

Everyone the author talked to at the VIGRE sites, those whose funding finished a few years ago, and those who are still getting funds, state that the changes made to the spirit of the department will not disappear. This spirit and culture supports students now and will continue to do so. In some cases, universities did not even recognize the change until after VIGRE funds stopped. This was particularly true at Princeton, where it took seven years, not the five of VIGRE funding, to put in place all of the ingredients necessary to integrate research and education up and down the ladder and for everyone to realize what had changed. Everything takes more time than one expects. This, too, is a lesson for funding agencies – continue to follow big projects after the funding stops to gain the greatest insights into what has occurred and what has been accomplished.

6.2 What is needed for implementation elsewhere?

The stories told here are examples of successful activities and approaches developed at VIGRE sites largely in response to a particular NSF program. Some of these activities, such as the Boot-camps for graduate students and early research experiences for undergraduate and graduate students have spread to other VIGRE sites. The books developed by and for Princeton University are now used at multiple sites. There is little or no reason why these very same activities and approaches can't be used in other university and college mathematical sciences departments. Adaptation of successful activities and approaches is both possible and extends the reach of the whole VIGRE program. Funding from other NSF programs may help when the expense is greater than the department can afford. The mathematical sciences workforce of the 21st century will come from colleges and universities of all sizes all across the country so all of them need to be engaged in meeting these needs.

What are the critical components necessary for the transferability of these success stories?

6.2.1 Early research experiences

As is indicated by the breadth of REUs offered throughout the country by colleges and universities of all types, it is possible to provide quality early research experiences for both undergraduate and graduate students anywhere there are faculty who are currently active researchers and have an interest in learning about open problems, possibly even problems outside their own area of expertise. The books written at Princeton University and UC Davis are available to assist all faculty in these endeavors.

Developing a breadth of interesting, appealing, and doable topics for student research is the most critical and demanding component of faculty efforts to provide early research experiences for undergraduate and graduate students. Students need a set of problems that they are ready for and are able to attack, so they avoid totally frustrating experiences. As Steve Miller of Brown University says, there are many necessary components for a successful undergraduate research class, ranging from motivated students and dedicated faculty, to adequate resources (computers and staff, etc.), to choosing a 'good' problem. What makes a good problem or project depends enormously on the faculty member and the students receiving the problem. For faculty, there is a strong urge to choose problems related to their current research, both because of their interest in the subject as well as their desire for a publication. With care it is often possible to find projects related to a faculty member's research and interests. However, there is an enormous danger in ending up with too technical a problem, or one with too extensive prerequisites, or one where the coding difficulties and computing issues make the problem computationally inaccessible for beginning students. In general, the best projects involve a problem which can be described to students without too many preliminaries, and one where the computational requirements are not overwhelming. Successful projects require matching a student's interest with those of the faculty mentor, taking into account the student's mathematical and programming background. Projects in matrix theory, combinatorics, statistics, and number theory often make good candidates.

Larger universities are able to set up research groups around specific topics and include other areas of the university as is being done at most of the VIGRE sites. These groups focus responsibility on more than one person by including faculty, post-docs and graduate students, as well as undergraduates, who generate problems for the whole group to attack. Research groups also have the advantage of having a range of expertise in the group, for example only one member has to be able to write code to help investigate the problem.

Since this is the first taste of research for many undergraduate and graduate students, mentoring and adequate supervision are essential. Mentoring often comes more naturally in the context of working on research on interesting topics. What matters most is that the student learns mathematics through his/her investigations, and is excited and motivated to learn more mathematics. This can take place at four year colleges as well as at major universities.

6.2.2 Mentoring

Strong mentoring was pervasive in all program aspects at successful VIGRE sites. Everyone had a role in mentoring someone else in strong mentoring programs, whether it was a faculty mentor of a graduate student, a senior undergraduate mentoring a freshman, or an industrial person mentoring an undergraduate mathematics student.

Mentoring in college and university mathematical sciences departments can grow out of traditional departmental advising programs. However, advising has become a central college function and is no longer based in the department in many instances. Because mentoring goes beyond advising about specific courses and graduation requirements, and engages students as well as faculty, it need not conflict with other college or university advising activities. Strong mentoring within a department creates a community of teachers and learners, and thus requires a commitment of the entire department, even if each individual is not a specific mentor to someone.

At VIGRE sites mentoring is a personal as well as professional relationship between two people that developed over a period of time. Effective mentoring is characterized by mutual trust, understanding, and empathy. It need not be time intensive, but it requires opportunities for communication, the ability to ask the right question and offer suggestions, and most of all careful listening. Mentors can change over time, especially when the relationship is not what either person is comfortable with, or productive in an intended direction.

No mentor can know everything about everything necessary for success, thus multiple mentors – ones of different ages, background, and personalities – are of great benefit. Mentoring within research groups is particularly successful because a student, undergraduate or graduate student, receives mentoring from students with more experience, post-docs, and faculty, all in one setting. This setting provides opportunities for personal and professional mentoring, about research, teaching, and life in general.

Mentoring programs can be effective in mathematical sciences departments at colleges and universities of all sizes across the country. It is one of the ingredients of vital mathematical sciences departments that can occur at departments whose primary focus is research and at those whose primary focus is teaching.

6.2.3 Transition Experiences at all Levels

Transitions at various points in a person's career are often the hardest, even for the most talented. Transitioning from undergraduate general studies to mathematical sciences major, from undergraduate to graduate student, from course taking to research in graduate school, from graduate student to post-doc, and from post-doc to faculty member are all difficult. Many of the project directors said that the VIGRE program made them aware of these transitions and the need to provide ways of helping students successfully maneuver their way through them. Many believe that the creation of these transition activities empowered students and faculty and caused a substantial change in the culture of the department.

The best example of an activity created to support students through a tough transition is the *Boot-Camp* for new graduate students, like WOMP at the University of Chicago. These *Boot-Camps* have been transported from VIGRE site to VIGRE site and are easily adapted to any university with a graduate program. The first step to initiating a *boot camp* for new graduate students is to recognize that these new graduate students took their undergraduate courses at many different colleges and universities and therefore their background is by no means uniform. The second step is to engage current graduate students along with faculty in designing and implementing a plan to help new graduate students with both the academic demands of graduate school and the environment in which they live. Current graduate students have already lived through the experience and know better than most the challenges of starting graduate school. The third step is to try out the plan and get feedback from the first round to reshape the plan to be even more effective the next year.

Early research experiences for both undergraduate and graduate students (REUs and REGs) facilitate the transition from course-taker to researcher. These activities have been described in the first section of this chapter. The inclusion of open ended or unsolved problems in the curriculum further facilitates the transition to research. As was noted before, a number of universities attach laboratories to specific courses to provide students with experience in the experimental nature of mathematical research. REUs, curriculum changes, and laboratories are possible options at four-year colleges and universities.

University departments often believe that by not requiring any teaching of post-docs they are freeing them to do more research, and thus become more successful on the job market. However, VIGRE University post-docs declared that the post-doc programs which included teaching components are and were critical to their transition to becoming successful researchers and teachers. They cited numerous incidents where their appointments to Assistant Professor positions were based on their teaching experience managing whole classes, as well as their research portfolio. The fact that they had been involved with VIGRE activities, such as REUs, high school outreach programs, research clusters and more was an added bonus.

6.2.4 Internships

The documents from the Council on Competitiveness, the Business-Higher Education Forum, and the National Academy of Sciences, all state the firm belief that internships are critical to building the scientific workforce of the 21st Century. However, many of the VIGRE sites indicated how hard it was to initiate and maintain an internship program even when they believed it was essential for them to do so.

To develop internships in the mathematical sciences at colleges and universities requires a number of fundamental steps:

1. *An awareness of the variety of jobs in the mathematical sciences.* Many faculty members are aware of a few places that hire mathematicians and statisticians like NSA, but they are not aware of the full range of opportunities available, such as at aerospace companies or federal laboratories. The Sloan Foundation has an excellent careers website called the *Sloan Career Cornerstone Center* with a component on the mathematical sciences. There are a number of national databases of internships that subdivide to the level of mathematics.
2. *Knowledge of companies and laboratories who hire interns in the mathematical sciences.* The most successful sites at developing internships, like the University of Colorado, had a history of internships in their department or in other departments on campus. Engineering and business colleges have a longer history with using internships and can help the mathematical sciences department identify companies that they work with that are likely to hire mathematical sciences interns. On campus, the Career Center often has sections devoted to paid and unpaid internships, and can provide information on where other departments have been successful, the minimum requirements companies look for, such as grade point average.
3. *Leadership for the internship component.* Everyone in the department plays a role in developing internships, but it requires one individual who is responsible for the overall endeavor. Faculty can and do invite speakers from industry who then become resources for internships. Alumni are often in the best position to recommend internships to the department and to their company or lab – the companies are interested in recruiting others with similar backgrounds. A solid departmental alumni tracking program exists in very few places, but is an essential part of any internship program.

Broadening the educational experiences of undergraduate and graduate students to include work experiences, motivates students in the mathematical sciences and gives them the opportunity to see the range of possible jobs available to them when they finish their degree. Students are enthusiastic about internship opportunities whether they are ultimately interested in an academic position or one outside of academia.

6.2.5 Community Involvement

Mathematical Sciences Departments are a part of a much larger community, one that includes other departments, colleges, academic centers, schools, corporations, and informal education centers. Drawing people from each of these areas into the work of the department enriches the department, and the department in turn enriches the greater community. This is true regardless of the size of the department, the type of college or university, or the interests of the faculty.

Activities like the Challenge of the Week are relatively easy to get started and reap numerous benefits. The hardest part is getting a faculty member or two to take responsibility for finding a problem for each week, posting it on a website, reviewing the entries, and naming a winner. Various colleges and universities can share their problems once they have been tried and solved.

Activities for teachers and students in schools not only helps fill the pipeline, but provides for better overall K-12 mathematics programs. Programs like the Young Scholars and Sesame at the University of Chicago which utilize undergraduates and graduate students in the program provide these people with opportunities to consider teaching at all levels. They bring a fresh enthusiasm to their work with students and teachers.

The more minds one can bring to the solution of real problems, the quicker a solution can be found. Interactions, like the ones between UCLA and IPAM and UW and PIMS, benefit students and faculty, as well as facilitate speedier solutions to tough problems. Involving researchers from laboratories and companies in the work of the department helps generate problems and their solutions with a broad impact. These same laboratories and companies then become a base for internships. Similarly, faculty from other departments, colleges, and universities working together, and sharing experiences with the mathematical sciences department, are much stronger than any one individually.

6.3 Effective practices

Effective practices for mathematical sciences departments have been discussed in the last two sections, as well as in the main portion of this book, but they bear repeating at the conclusion of this book. The hard work, experience, and successes of the VIGRE departments have allowed one to generate a set of ten effective practices, which can significantly contribute to the success of mathematical sciences departments, students, and faculty in education, research, and ultimately the work place. Each of the VIGRE sites included most of these effective practices as part of their projects, if not at the beginning, then at the end. The practices are:

1. Move from a single student advisor format to one of mentoring where faculty mentor students at all levels, upper level students mentor lower level students, and both graduate and undergraduate students cross mentor each other. Mentoring goes beyond simply advising on courses and program details, and includes both personal and professional mentoring to create a community of teachers and learners.
2. Recognize that everyone has something to contribute to a department's efforts for quality education and be inclusive in implementation of new ideas and activities. Undergraduate students and graduate students can take on many responsibilities for determining what is needed and how to fill the need. They can help share the load of the department.
3. Promote broad-based research groups, which include undergraduate and graduate students, post-docs, and faculty. Faculty from other departments, centers, and colleges can appropriately participate in these research groups.

4. Further develop opportunities for high school students, undergraduates, and early graduate students to participate in concentrated research programs, such as REGS, REUs, and Young Scholars. These programs can include local students or go beyond the university geographic area.
5. Realize that moving from student to researcher and student to the workforce is hard and require the department to provide mechanisms for paving the way. Each stage of development as undergraduate, graduate student, post-doc, or faculty member, has its own set of challenges, but moving from one stage to the next is even harder. Boot camps, early research experiences, internships, are only a few of the examples given that soften these transitions.
6. Include the whole community, inside and outside the university, in the scope of efforts of the mathematical sciences department. Wider mathematical awareness and participation ultimately infuses energy in the mathematical sciences department and its activities.
7. Consider the whole range of potential careers for mathematical scientists when designing curriculum and internship opportunities, from providing open ended (research) questions in the earliest courses, to allowing for cross-disciplinary options, and to developing relationships with potential employers. The demand for people with PhDs and other degrees in the mathematical sciences will continue to increase, both inside and outside of academia.
8. Provide a full set of experiences in teaching, research, and departmental activities for post-docs. Post-docs appreciate the opportunity to teach a broad range of classes, and to be involved in new and ongoing initiatives, as part of their training for entry into the workforce.
9. Realize that leadership can occur where you least expect it. Shared leadership keeps initiatives going long after any one person leaves, and prevents the common burn-out factor from occurring.

10. Enjoy the changes in culture that occur and seeing more students successfully move through the stages of development. Enthusiasm comes from enjoying what you do and seeing the results of your labors.

6.4 Final conclusion

In the last analysis, VIGRE programs in mathematical sciences departments at universities around the Nation are to be applauded for the number of positive things they accomplished:

- They integrated research and education activities into a single unit; they are not disjoint pieces.
- They changed the way people in mathematical sciences departments interact with one another. Everyone is taken seriously and able to contribute, and connections are personal as well as professional.
- They provided a way to make researchable questions accessible to students at different stages of their development, bringing new excitement and interest to mathematical study. They found that questions drive the interactions, not courses.
- They empowered individuals at all levels to be creative and to develop innovative activities for the betterment of all.

These VIGRE mathematical sciences departments view the NSF investment as money well spent in terms of gains achieved. The examples of successes cited in this book are only a small sample of all of the successes to be found at VIGRE universities, but they serve to provide a clear legacy for the future! There is a proof of concept – can this concept be expanded to more places and sustained at all VIGRE sites? Ultimately, it must be determined if there is a real noticeable difference in the mathematical sciences workforce in ten to fifteen years as a result of changes made today at VIGRE sites. Has there been an impact on how students approach their own careers – are they better teachers, researchers, workers?

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The cover image is an adaptation from one used by the University of Colorado VIGRE.

APPENDIX

VIGRE Awards

Institution		Department(s)		Award Dates
University of Colorado – Boulder		Applied Mathematics		1999
Columbia University		Mathematics		1999
Princeton University		Mathematics, Applied & Computational Mathematics		1999
Pennsylvania State University		Mathematics		1999
Harvard University		Mathematics		1999
University of Washington		Applied Mathematics, Mathematics, Statistics		1999, 2004
North Carolina State University		Statistics		1999, 2004
University of Wisconsin – Madison		Mathematics		1999, 2004
Rutgers University - New Brunswick		Mathematics		1999
University of California – Berkeley		Mathematics		1999
Carnegie Mellon University		Statistics, Mathematical Sciences		1999
University of Arizona		Mathematics, Applied Mathematics		1999, 2006
University of Michigan - Ann Arbor		Mathematics		1999
Texas A&M University		Mathematics		2000
Brown University		Mathematics, Applied Math		2000
University of Chicago		Mathematics		2000, 2005
University of California - Los Angeles		Mathematics		2000, 2005
University of Illinois at Urbana - Champaign		Mathematics		2000

New York University - Courant Institute		Mathematics		2000
Rensselaer Polytechnic Institute		Mathematical Sciences		2000
Duke University		Mathematics		2000
Purdue University		Mathematics, Statistics		2000
Cornell University		Mathematics		2000
State University of New York - Stony Brook		Mathematics, Applied Mathematics & Statistics		2000
University of Illinois - Chicago		Mathematics, Statistics, Computer Science		2000
Yale University		Mathematics		2000
Iowa State University		Statistics		2001
University of Georgia		Mathematics		2001
Indiana University		Mathematics		2001
University of Texas – Austin		Mathematics		2001
University of Utah		Mathematics		2001, 2006
Ohio State University		Mathematics		2002
University of California – Davis		Mathematics		2002, 2007
University of California - Berkeley		Statistics		2002, 2007
Georgia Institute of Technology		Mathematics		2002
Carnegie Mellon University		Statistics		2003
University of Maryland		Mathematics		2003
Rice University		Mathematics, Statistics, Computational & Applied Mathematics		2003
Tulane University		Mathematics		2003
Stanford University		Statistics		2005
University of Iowa		Mathematics		2006

This material is based upon work supported by the National Science Foundation under DMS Grant No. 063728. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author and do not necessarily reflect the views of the National Science Foundation.